Power and Sample Size Calculation

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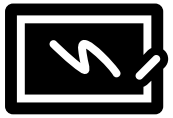
During the workshop

Ask short questions or clarifications during the workshop (either by Zoom chat or verbally). There will be breaks during the workshop for longer questions.

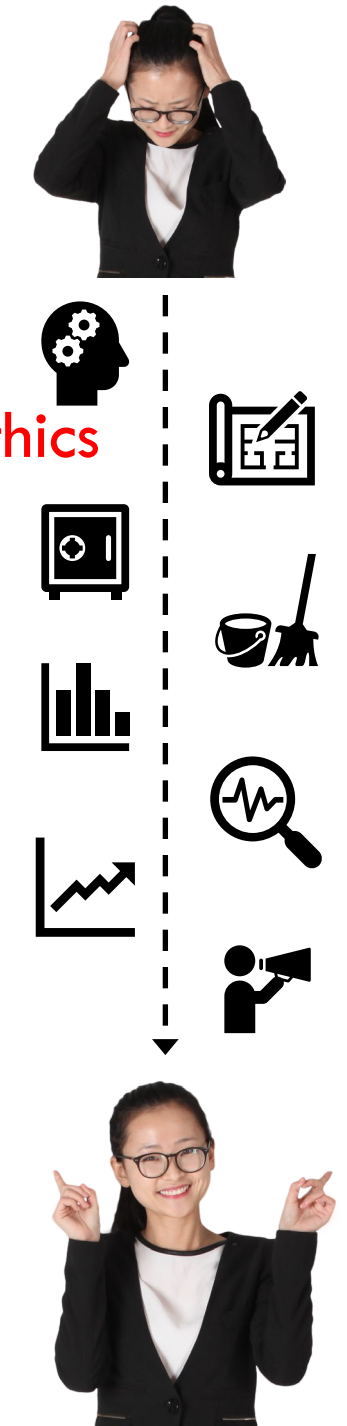


Slides with this blackboard icon are mainly for your reference, and the material will not be discussed during the workshop.

Challenge questions will be encountered throughout the workshop.

General Research Workflow

1. **Hypothesis Generation** (Research/Desktop Review)
2. **Experimental and Analytical Design** (sampling, **power,** ethics approval)
3. **Collect/Store Data**
4. **Data cleaning**
5. **Exploratory Data Analysis (EDA)**
6. **Data Analysis aka inferential analysis**
7. **Predictive modelling**
8. **Publication**



Outline

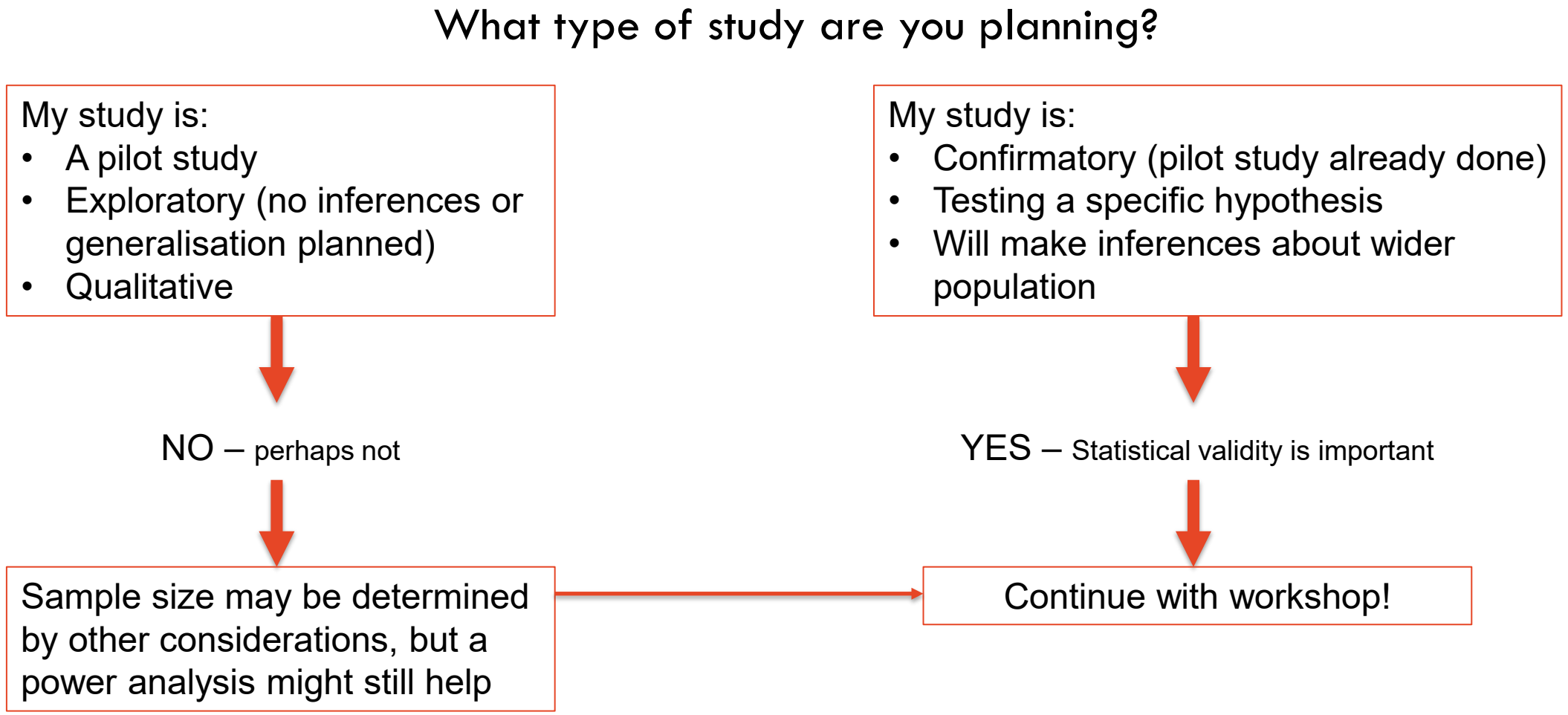
- Statistical power and sample size calculation - concepts
- Software tools – Statulator, Power and Sample Size (PS)
- Example A: Difference between two means (t-test)
- Example B: Difference between two means (Mann-Whitney)
- Example C: Difference between two proportions (z-test)
- Example D: Estimation of a single proportion
- Power calculation for other designs
- References

Why do we need to calculate power and sample size?

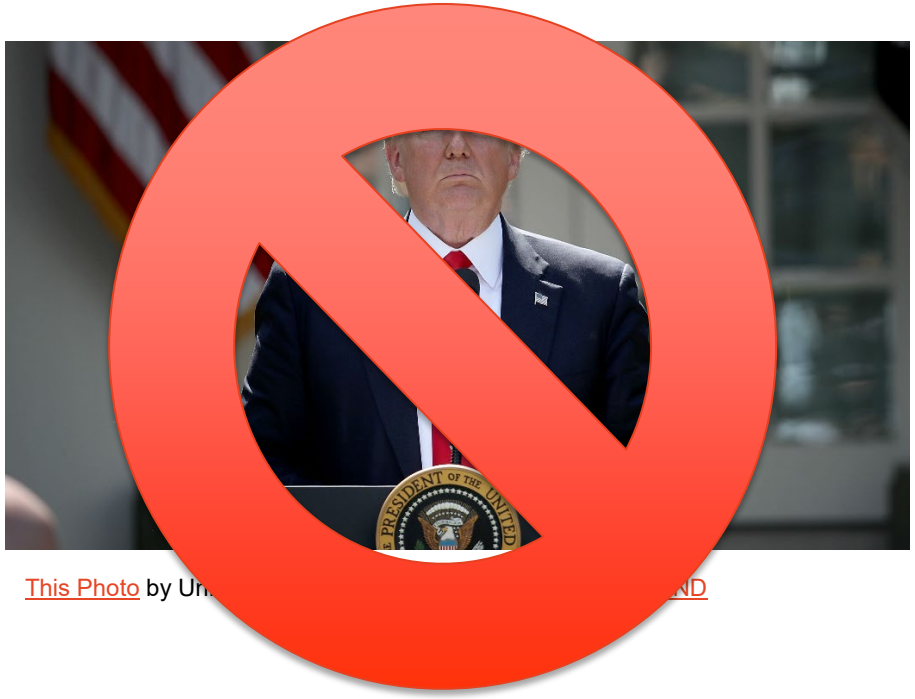
Why do we want to estimate the power of a study?

- To know that it is worth doing the study
- …and make sure we are not wasting our time
- To plan the time and resources necessary
- To get a grant application approved
- To make sure the study design is ethically acceptable

But do I <u>really</u> need to calculate power?



What is the power of a study design?



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Statistical power: The power to know…

What do we want to know?

- Statistics involves using a sample to estimate of a population property (a mean, a mean difference, an odds ratio, or some other property).

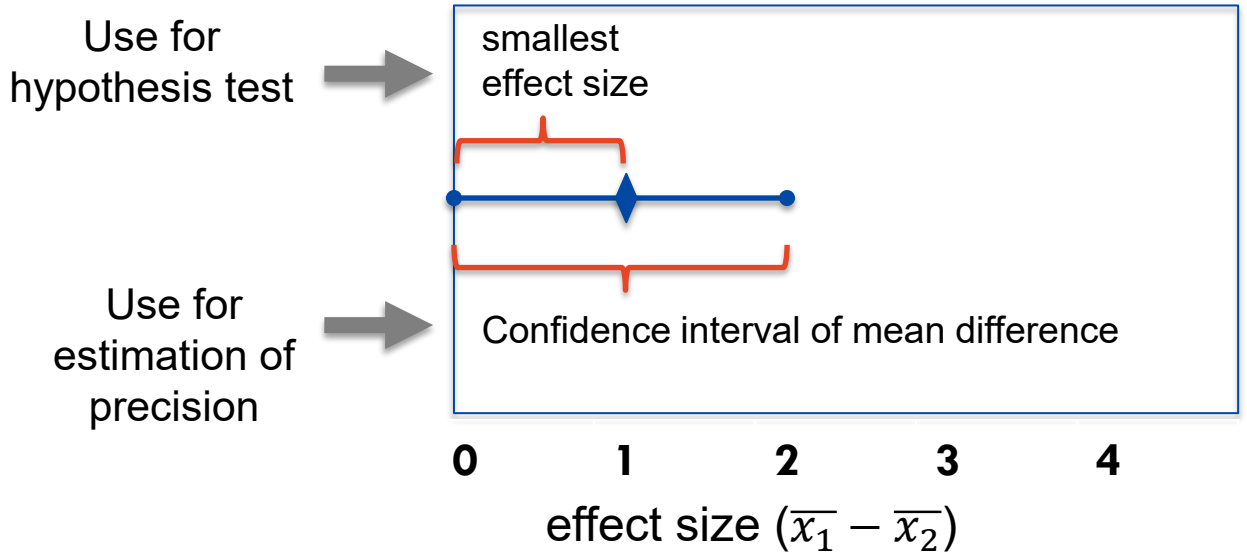
- Usually we want to perform some inference: a hypothesis test that our property takes a particular value (e.g. drug X and drug Y mean difference = 0)

- Sometimes, we only want to estimate the population property and not perform inference (e.g. the prevalence of disease A in the population is 10.5% [9%-11%])

What do we want to know?

- Sample sizes can be calculated for either research goal

- In this workshop, examples A-C are based on hypothesis tests of two groups, while example D is based on estimation of a proportion to a specified level of precision



Further reading on the use of CI for sample size calc: see chapter 3 of **"Determining Sample Size Balancing Power, Precision, and Practicality"** by **Dattalo**

We propose an alternative hypothesis but test a null hypothesis

Start with the hypothesis that you have generated, for example:

"Novel drug X lowers blood pressure more than standard-of-care drug Y"

In statistics, this is referred to as the alternative hypothesis (H_1). This is the hypothesis we are interested in. Classically, we test the veracity of the null (H_0) hypothesis:

"Drug X and drug Y lower blood pressure by the same amount"

A statistical test of the null hypothesis is always subject to **uncertainty.**

Errors can't be avoided, but can be controlled

Despite this **uncertainty,** when we perform a null hypothesis test, we are setting up a binary choice that can result in making a correct decision, or an error

We can't completely avoid errors, but we can choose the rate of error that is acceptable to us given **uncertainty**

We make decisions based on hypothesis testing

Null hypothesis is in reality

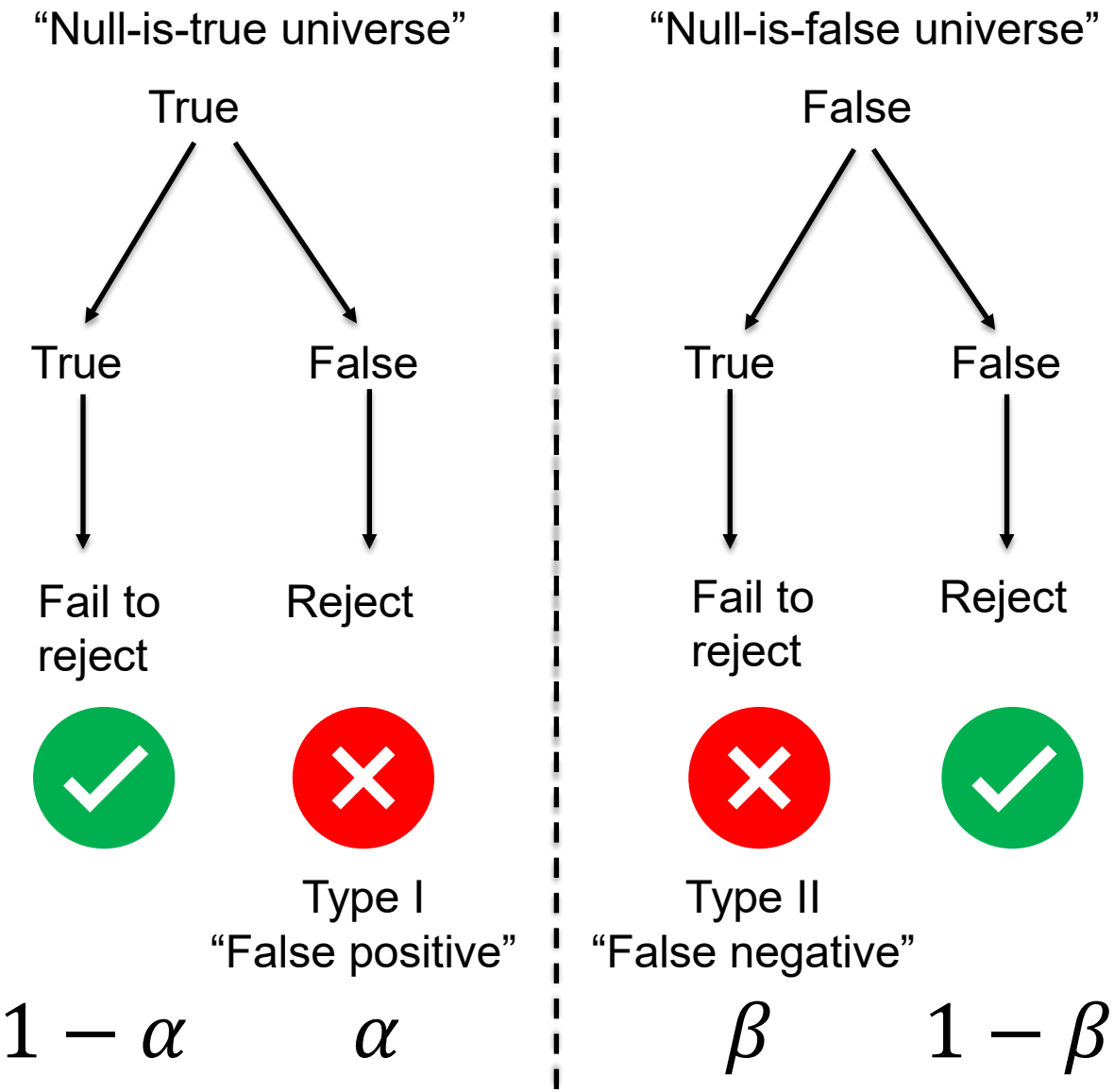
We conduct our hypothesis test

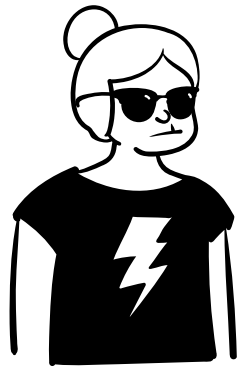
What do we think about the null hypothesis?

What do we decide about the null hypothesis?

Did we make the correct decision?

How frequent (how probable) is this outcome?





Statistical power!

Alpha and beta are our chosen error rates

Type I error "False positive"

- Rate of false positives designated by the signficiance level, α
- We want the rate of false positives to be as low as possible
- The *convention* is to set the significance level to $\alpha = 0.05$, at this rate, we accept that even when the null hypothesis is true, it will be rejected one in every twenty runs of a study

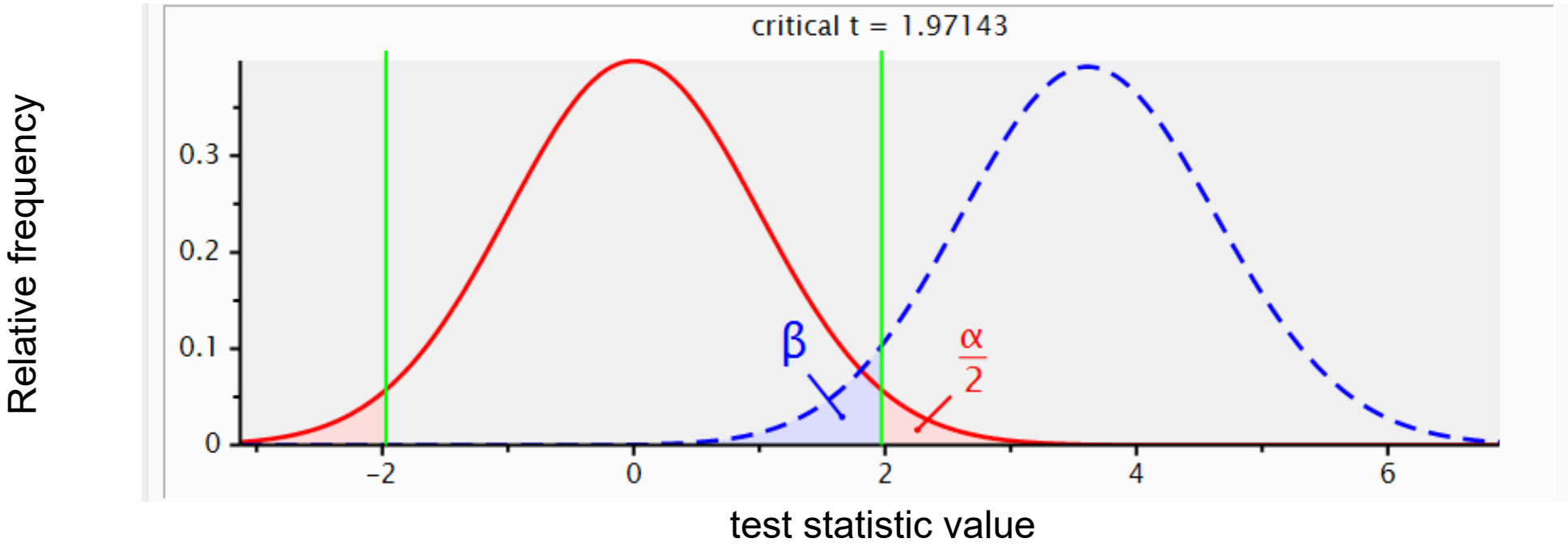
Type II error "False negative"

- Rate of false negatives designated by β
- Statistical power is the complement of β, denoted by $1 - \beta$
- We want power to be as high as possible
- The *convention* is $1 - \beta \geq 0.8$, at 0.8, even when the null hypothesis is false, it will not be rejected one in every five runs of an experiment

P-values allow us to make decisions and quantify evidence

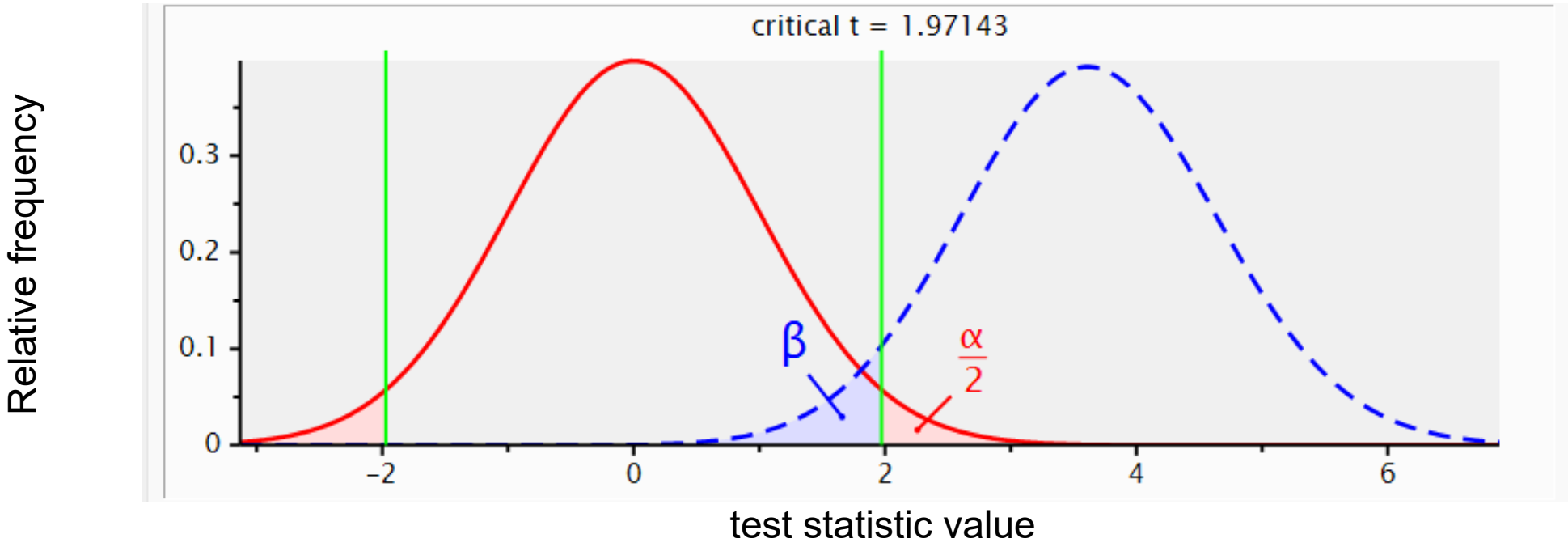
- Maligned, misunderstood and occasionally spurned, p-values have a simple purpose: to quantitate how probable the null hypothesis is given some assumptions
- Remember that p-values are produced from a "null-is-true" perspective, so we always assume that the null hypothesis is true when calculating them
- If a p-value falls below the significance threshold α (and especially if it falls far below a given threshold), we may suspect that we are in fact in the "null-is-false universe" and reject the null hypothesis
- Counterintuitive: We don't test the alternative hypothesis directly, so the smaller the p-value, the more plausible the alternate hypothesis becomes
- A discussion of how p-value thresholds relate to α and β is given on the following slides for your reference

Alpha is also the p-value threshold



- Here is a plot from G*Power software
- The red curve shows how a given test statistic (a t-statistic in this case) is distributed in the "null-is-true universe", where the null hypothesis is always true
- The blue and dotted curve shows how this test statistic is distributed in a "null-is-false universe" for a particular effect size – more on effect sizes later
- The chosen α determines the critical value of the test-statistic (green vertical line)

Alpha is also the p-value threshold



- For the usual two-tailed hypothesis, the α area is distributed across the two tails of the red solid line distribution (called the null distribution). The total area is equal to α
- A p-value is calculated based on the area of the null distribution that covers test statistic values more extreme than that observed (on both sides for a two-sided test)
- There are no overlaps between the α and β areas, reflecting the binary choice made of whether to reject the null hypothesis or not
- 1- β is the area under the blue dotted distribution that is not shaded

Building your intuition of a deeply unintuitive procedure

- For a more interactive view of these distributions to build your intuition check out: https://rpsychologist.com/d3/nhst/
- Note that the author is "deeply skeptical about the current use of significance tests", but null-hypothesis statistical testing (NHST) is a mainstay of modern research. So, we must know how to use it even if we don't like it.

Not every hypothesis is a superiority test

The most recognised and widely-used hypothesis test is whether two measures are *exactly* equal or different by any amount (a superiority test).
H_0: Difference = 0
H_1: Difference ≠ 0

Other study objectives will lead to other types of hypothesis test.

The types below are frequently found in clinical trials (e.g. a novel drug performs *no worse* than the existing drug: Non-inferiority test)

Similar tests:
- Equivalence test
- Minimum Effect test

See reference for further details: Julious, Steven A. Sample Sizes for Clinical Trials . Boca Raton: CRC Press/Taylor & Francis, 2010. Print.

We need inputs to calculate statistical power

- It will depend on:
  - Sample size
  - Chosen significance level (α)
  - Minimum effect size to detect
  - Variance within groups
  - Experimental design and type of statistical hypothesis test

Decisions regarding the study design can be critically important in determining statistical power. This is covered in the "**Experimental Design**" workshop.

- Usually, we want to calculate a sample size given a required minimum power.

Power calculation can become sample size calculation

- It will depend on:
  - ~~Sample size~~
  - Chosen significance level (α)
  - Minimum effect size to detect
  - Variance within groups
  - Experimental design and type of statistical hypothesis test
  - Statistical Power (1-β)

- Usually, we want to calculate a sample size given a required minimum power.

Decisions regarding the study design can be critically important in determining statistical power. This is covered in the "**Experimental Design**" workshop.

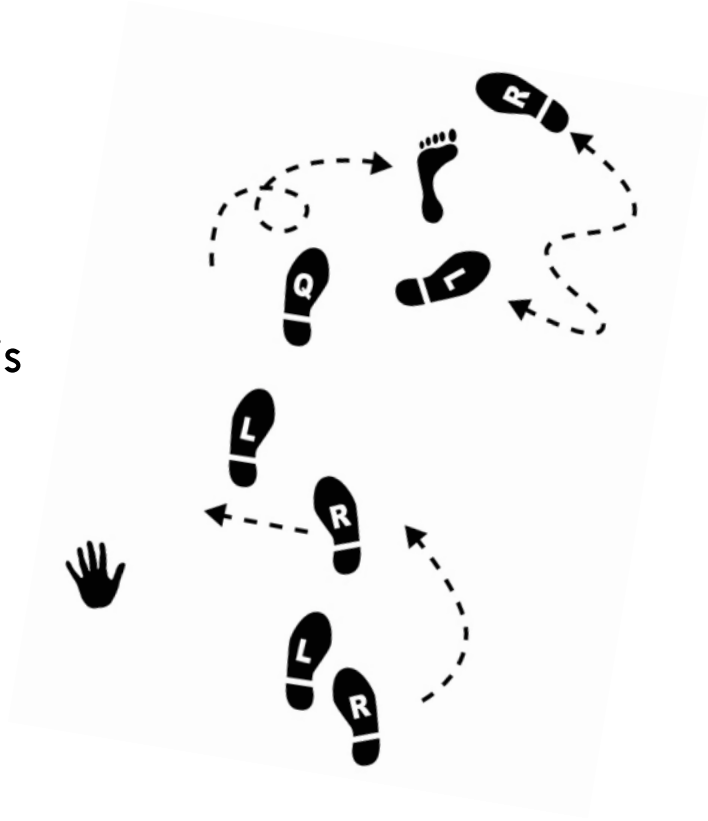
What is a workflow?

- Every statistical analysis is different, but all follow similar paths. It can be useful to know what these paths are

- We have developed practical, step-by-step instructions that we call '*workflows*', that can you can follow and apply to your research

- We have a general research workflow that you can follow from hypothesis generation to publication

- And statistical workflows that focus on each major step along the way including **study design and power**



Statistical Workflows

- Our statistical workflow can be found within this workshop
- Statistical workflows are software agnostic, in that they can be applied using any statistical software
- There are accompanying software workflows that show you how to perform the sample size statistical workflow using software packages:
  - **Online calculators: Statulator and PS**
  - **G*Power**
  - **SPSS**

Sample size calculation statistical workflow

Sample size calculation workflow steps

1. Determine experiment type and statistical test
2. Set α and $1 - \beta$
3. Set the smallest effect size of interest
4. Estimate the variance
5. Calculate the minimum sample size
6. Explore scenarios

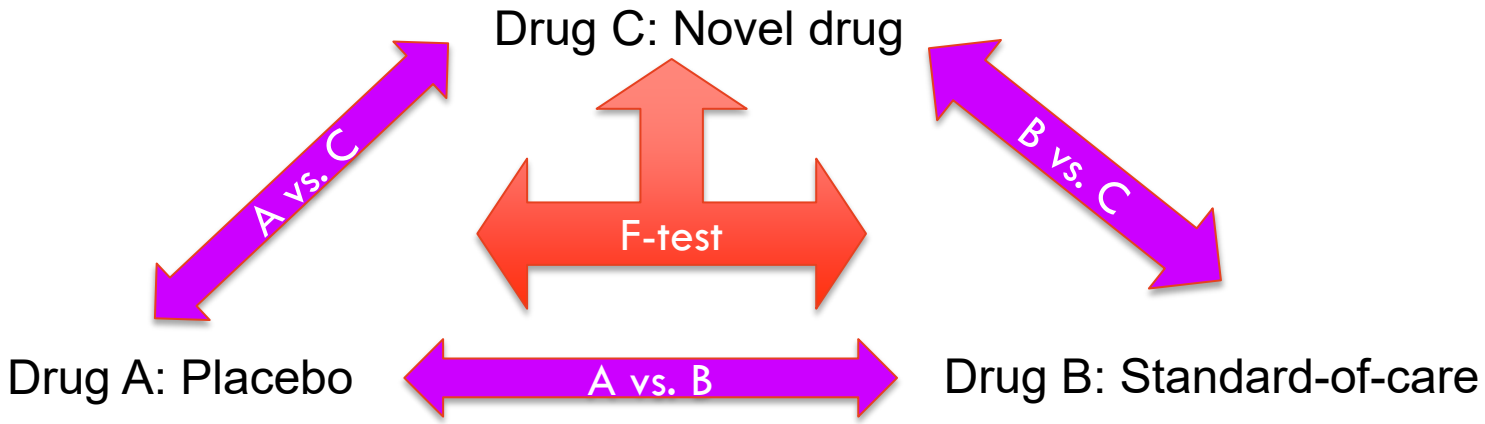
1. Determine experiment type and statistical test

For example:

Experimental design	Main assumptions	Proposed statistical test
Comparison of 2 means	independent groups, normally distributed outcome	t-test
Comparison of 2 means	independent groups, no assumption of normality	Mann-Whitney U test
Comparison of 2 proportions	independent groups	z-test/Fishers exact
Comparison of means, more than 2 groups	independent groups normally distributed outcome	ANOVA, F-test

1. Determine experiment type and statistical test

- Tip from the consulting room:
  - Your study design may lead to a series of hypothesis tests of interest
  - You need to choose which of these hypothesis tests should be used to "power your experiment"
  - A typical example is an ANOVA design experiment
  - With three groups this would result in four hypothesis tests: an F-test, and three post-hoc t-tests



Which of these tests should be used to power the experiment (i.e. ensure sufficient sample size)

2. Set α and $1-\beta$

Setting values of parameters

- Typically choose $\alpha = 0.05$ (or lower)
- Typically choose power $(1-\beta) = 0.8$ (or higher)

You should have a justification for choosing a particular α and $1-\beta$. There is no reason why the conventional values must be used… consider the two examples on the next slide.

2. Set α and $1-\beta$

Let's consider a couple of scenarios:

- You are investigating the rate of detection of a pathogen in hospitals that can be fatal in immunocompromised patients. It is thought that the pathogen is very rare, but you suspect it is more common. Your study is very expensive to set up, but it is relatively inexpensive to collect more samples. If there is a negative finding, it is unlikely the study will be repeated. Should you use the conventional power ($1-\beta$) of 80%?
  - *Consider increasing $1-\beta$ (decreasing β) as 80% means a one in five chance of a false negative even if the alternative hypothesis is true*

- You are investigating the use of novel disinfectants on hospital pathogens by screening a panel of potential disinfectants. If a disinfectant is found to be effective, the disinfectant will be tested again in a new and larger sample. Should you use the conventional α of 0.05?
  - Consider increasing α. *A false positive is less of a problem, because the finding will be replicated*

2. Set α and $1 - \beta$

A good example of how a personal preference of an influential figure can become an iron-clad convention.

"It is proposed here as a convention that, when the investigator has no other basis for setting the desired power value, the value .80 be used. This means that β is set at .20. This arbitrary but reasonable value is offered for several reasons (Cohen, 1965, pp. 98-99). ***The chief among them takes into consideration the implicit convention for α of .05.*** *The β of .20 is chosen with the idea that the general relative seriousness of these two kinds of errors is of the order of .20/.05, i.e., that Type I errors are of the order of four times as serious as Type II errors.* ***This .80 desired power convention is offered with the hope that it will be ignored whenever an investigator can find a basis in his substantive concerns in his specific research investigation to choose a value ad hoc."***

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed). L. Erlbaum Associates.

Multiple testing

- In high-throughput biology, which involves many performing many statistical tests for each outcome being measured, type I error is often controlled using a False Discovery Rate (FDR) rather than the usual Family-Wise Error Rate (FWER). This is effectively setting the α to a much higher level. Any differences detected on the high throughput platform will be verified using a lower throughput technology.

- A common and familiar form of multiple testing is adjusting for multiple post-hoc tests following an ANOVA F [omnibus] test

- Bonferroni is the most stringent multiple testing correction, but it easy to apply

- Other forms of multiple testing correction can be applied in other contexts. Come to a consult for further guidance.

3. Set the smallest effect size of interest

Effect size e.g.s: difference in means, difference in proportions, odds ratio…

What is the smallest effect size of interest?

- Decide on a smallest effect size of interest (SESOI) as a target to detect in your hypothesis test. This should be based on the smallest effect size that is of *scientific interest/significance*

- You will need to use your domain expertise to decide this, e.g.:
  - What constitutes a *clinically* meaningful difference between patient groups in a score measuring depression?... What is a *biologically* meaningful difference in gene expression in two different cell types?... What difference in number of seconds to complete a task would you consider important in a mouse *behaviour* model?

- If it is unknown, you may need to consider a range of plausible effect sizes and evaluate the required sample size as each of these (as described in the examples)

3. Set the smallest effect size of interest

too small

Effect size chosen is smaller than necessary

- The sample size is larger than necessary
- Possible waste of resources
- Can achieve statistical significance with an effect that is too small to be interesting or useful

just right

Effect size chosen is based on sesoi

- The sample size is just right
- If statistical significance is achieved, then it will align with scientific significance
- Most efficient use of resources

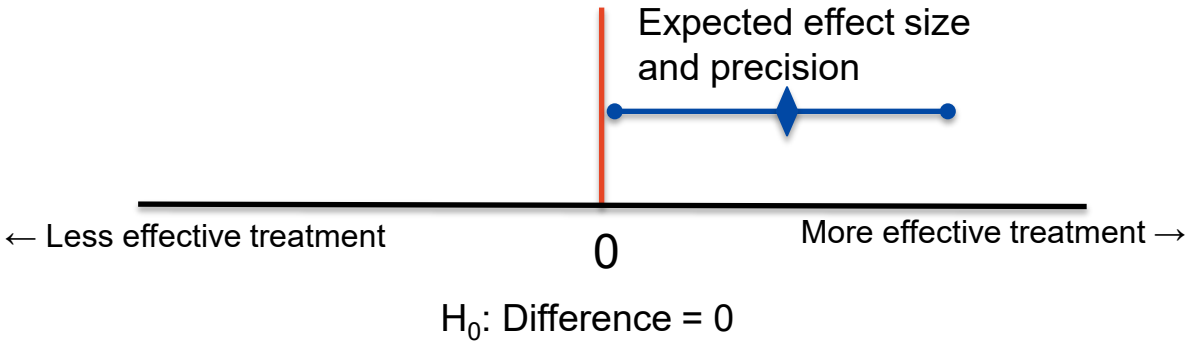
too large

Effect size chosen is larger than necessary

- The sample size is too small
- Not able to achieve statistical significance for small effect sizes of interest
- May detect large effects only
- Possible waste of resources

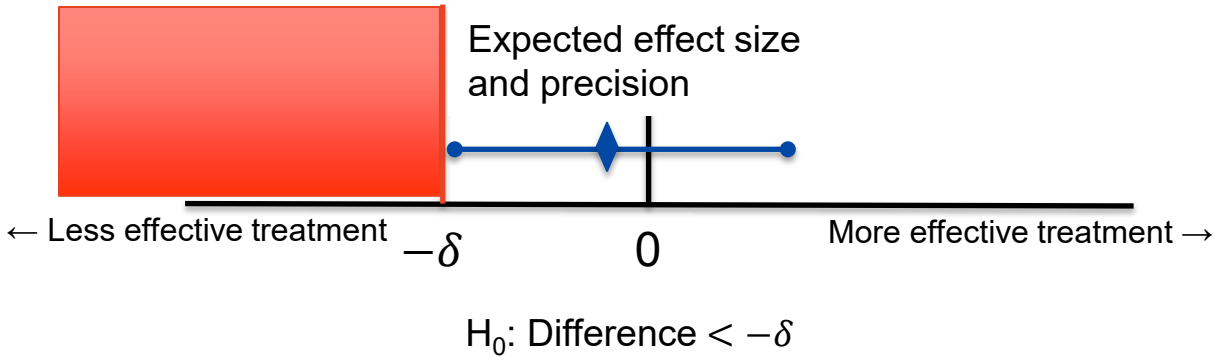
goldilocks

3. The SESOI may not be the only effect size you need



Effect size – note that your SESOI may not be the same as your expected effect size. Depending on the hypothesis test you may need to think of these as different quantities:

For the typical test (superiority), you use only your SESOI to calculate the sample size (statistical significance and scientific significance align)



For a non-inferiority test, your SESOI determines delta (δ): the boundary of the range of effect sizes that have no clinical/practical/scientific significance.

You also need an expected effect size to calculate the sample size.

https://lakens.github.io/statistical_inferences/09-equivalencetest.html

4. Estimate the variance

Within study variance is often the big unknown in a sample size calculation

How to estimate it?

- Estimate standard deviation (or proportions) from previous experiments?
- Consider theoretical bounds (eg proportions, for 5pt scales)
- Seek expert knowledge?
- If no idea, may be best to do pilot study

4. Estimate the variance Standardised Effect Size

Alternative: Use the Standardised Effect Size

Many effect sizes can be "standardised" by considering the ratio of the effect size to a within group standard deviation.

For example: Cohen's d is the ratio of the difference in means to the pooled standard deviation

$$d = \frac{\overline{x_1} - \overline{x_2}}{s}$$

Cohen's d is therefore analogous to the number of standard deviations difference, or the z-score difference. Also called the standardised mean difference (SMD).

Cohen's d is used within the popular G*Power software

4. Estimate the variance Standardised Effect Size

Alternative: Use the Standardised Effect Size

Instead of deciding on effect size and an estimate of SD, we can choose a value of Cohen's d based on accepted interpretations of relative size.

Effect size	*d*	Reference
Very small	0.01	Sawilowsky, 2009
Small	0.20	Cohen, 1988
Medium	0.50	Cohen, 1988
Large	0.80	Cohen, 1988
Very large	1.20	Sawilowsky, 2009
Huge	2.0	Sawilowsky, 2009

Other guidelines are published for other standardised effect sizes.

Note however that interpretation can vary across different fields of study.

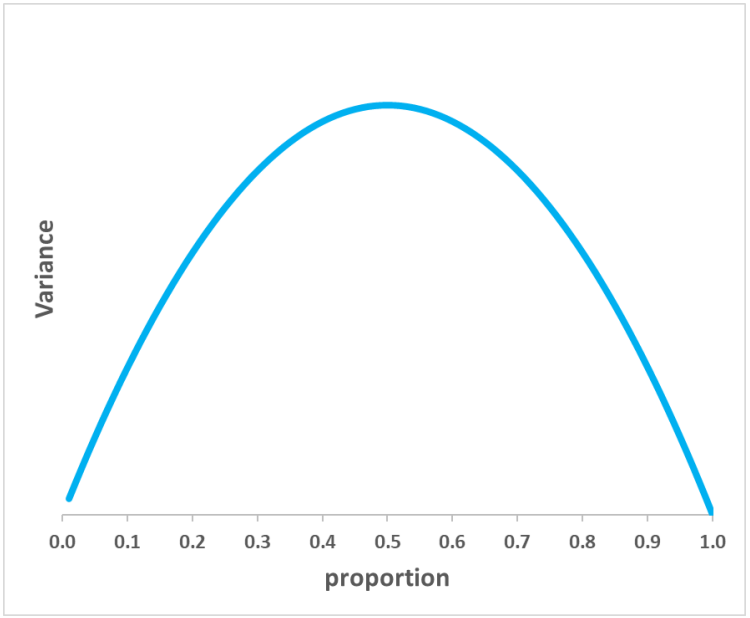
https://en.wikipedia.org/wiki/Effect_size#Cohen's_d

4. Estimate the variance Theoretical upper bound

For proportions the maximum variance occurs when p=50% and is at a minimum when p=0% and 100%.

So we can use p=50% to find a theoretical upper bound.

$$Variance(p) = p(1-p)$$



Why is this the case?

It might be strange to think about the variance of a proportion. You can think about it as a feature of a binomial (0 or 1) outcome. What proportion of subjects are 0 and how many are 1 for some outcome? (e.g. 0 = non-smoker and 1 = smoker)

Proportions must be between 0 and 1. When our estimate is close to these limits, there is less variability in our estimates for the same number of subjects surveyed.

4. Estimate the variance Theoretical upper bound

For ordinal responses such as 5pt scales a similar limit applies:

Possible responses are: 1, 2, 3, 4 or 5

Mean=3 Min=1 Max=5

$$Variance(5pt\ scale) = (max - mean)(mean - min)$$

$$Max\ Variance(5pt\ scale) = (5 - 3)(3 - 1) = 4$$

In practice the actual variance will be smaller than the max. A rule of thumb is explained on StackExchange

https://stats.stackexchange.com/questions/23519/how-do-i-evaluate-standard-deviation

5. Calculate the minimum sample size

- This is typically done using a software package
- Formulae for the calculation vary with the type of experimental design and the statistical test. We won't look at these too closely, but let's note some common features to reinforce the theory we have learned.
- The formula below is for a difference in means of two groups, where the standard deviation is known (good for illustration, though not often used in practice).

$$n = \frac{2\,(Z_{\alpha/2} + Z_{1-\beta})^2}{\left(\frac{\mu_1 - \mu_2}{\sigma}\right)^2}$$

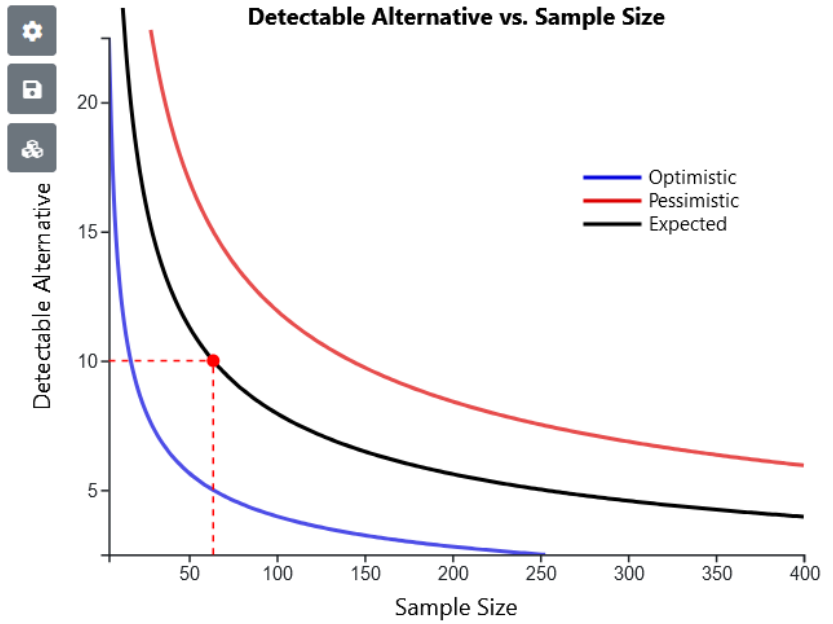
The Z are critical values based on the chosen α and β. The smaller α or β are, the larger their critical values.
Note that for a fixed sample size if we increase alpha (accept higher FP) rate, we would increase power $1 - \beta$

Note that the expected difference in means $\mu_1 - \mu_2$ is divided by the expected SD σ. The denominator resembles Cohen's D, the standardised effect size. We use the greek letters here instead to indicate these are 'known' values.
The whole expression is squared. The smaller the expected difference, the larger the sample size required to detect it.

6. Explore scenarios

Tip from the consulting room:

- Don't just calculate a single sample size n for a **power calculation**
- Use the software to calculate n for a range of scenarios to explore uncertainty in the input values used in the calculation. You may have a large uncertainty in the variance, and/or the expected effect size.
- A **Power Analysis** incorporates informative plots to design and plan your study



Useful for experiment planning: *Consider also the shape of the cost curve for sample data collection*
In the above example increasing sample size up to ~100 yields big effect size detection benefit but increasing sample size beyond ~100 yields diminishing returns. You can use this information in combination with feasibility considerations to choose a target sample size.

Examples using Statulator and PS

We will work through four simple examples:

A: Difference between two means (continuous response)

B: Difference between two means (survey response)

C: Difference between two proportions

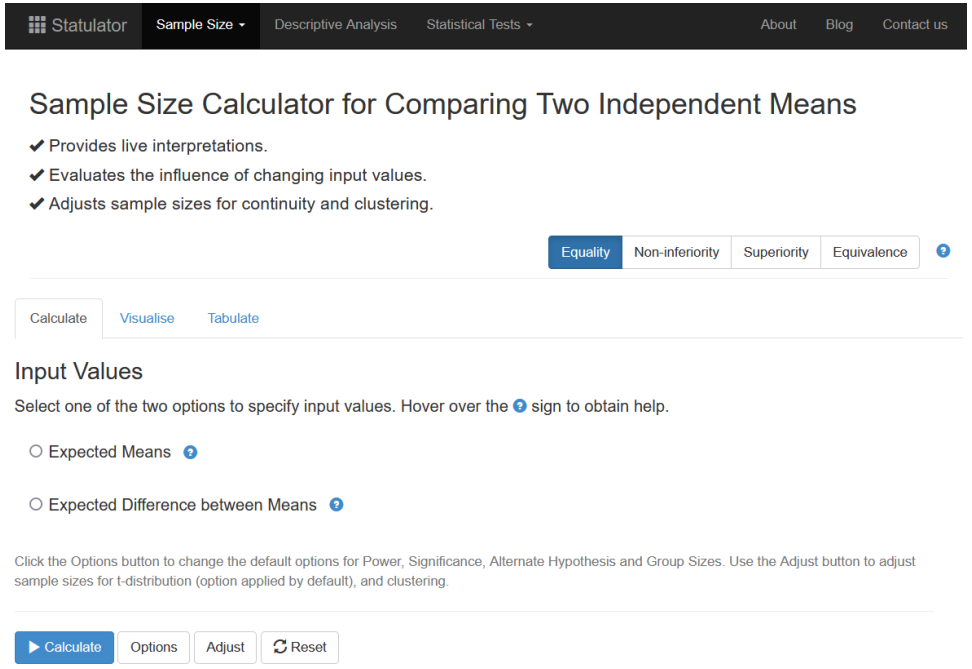
D: Estimation of a single proportion

Followed by a discussion of what to do when your study is more complicated than this

Power calculation software used in this workshop

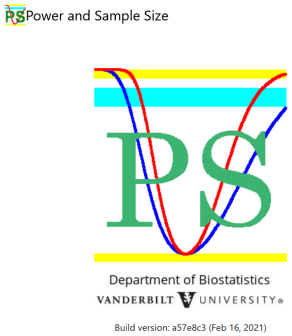
Statulator

- Free on-line statistical calculator
- Developed by epidemiologists and biostatisticians at Sydney University
- Easy to use
- Live interpretation provided for each calculation
- Visualisations to help you explore scenarios
- Incorporates other types of hypothesis test beyond the usual superiority type



Power and Sample Size (PS)

- Free on-line power and sample size calculator
- Useful for power analysis plots
- Live interpretation provided for each calculation



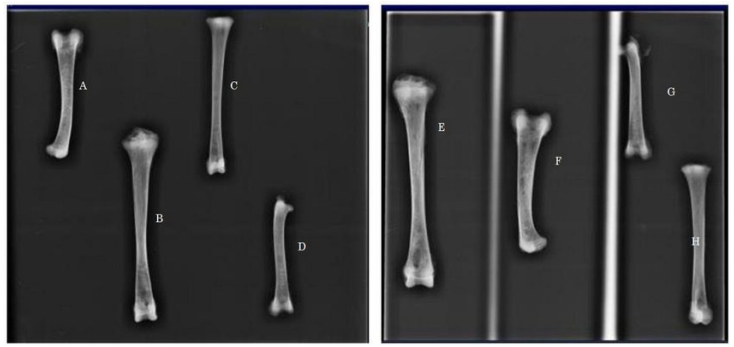
A. Difference between two means

Example: Chicken Welfare – Bone density

The bone density of chickens is an important indication of their welfare. We want to test to see if (mineral) bone density can be improved from 120 to at least 130 mg/cm^3

Control Group (1) = normal diet
Treatment Group (2) = high mineral diet
Response variable: Measure the tibia bone density after 6 weeks growth.
How many chickens do I need to detect a difference in bone density of 10 mg/cm^3?

What type of statistical test will we perform?



TY - JOUR AU - Mabelebele, Monnye AU - Norris, Dannah AU - Siwendu, Ndyebo AU - Ng'ambi, Jones AU - John, Alabi AU - Mbajiorgu, C.A. PY - 2017/01/01 SP - 1387 EP - 1398
T1 - Bone morphometric parameters of the tibia and femur of indigenous and broiler chickens reared intensively VL - 15 DO - 10.15666/aeer/1504_13871398 JO - Applied Ecology and Environmental Research ER -

A. Difference between two means

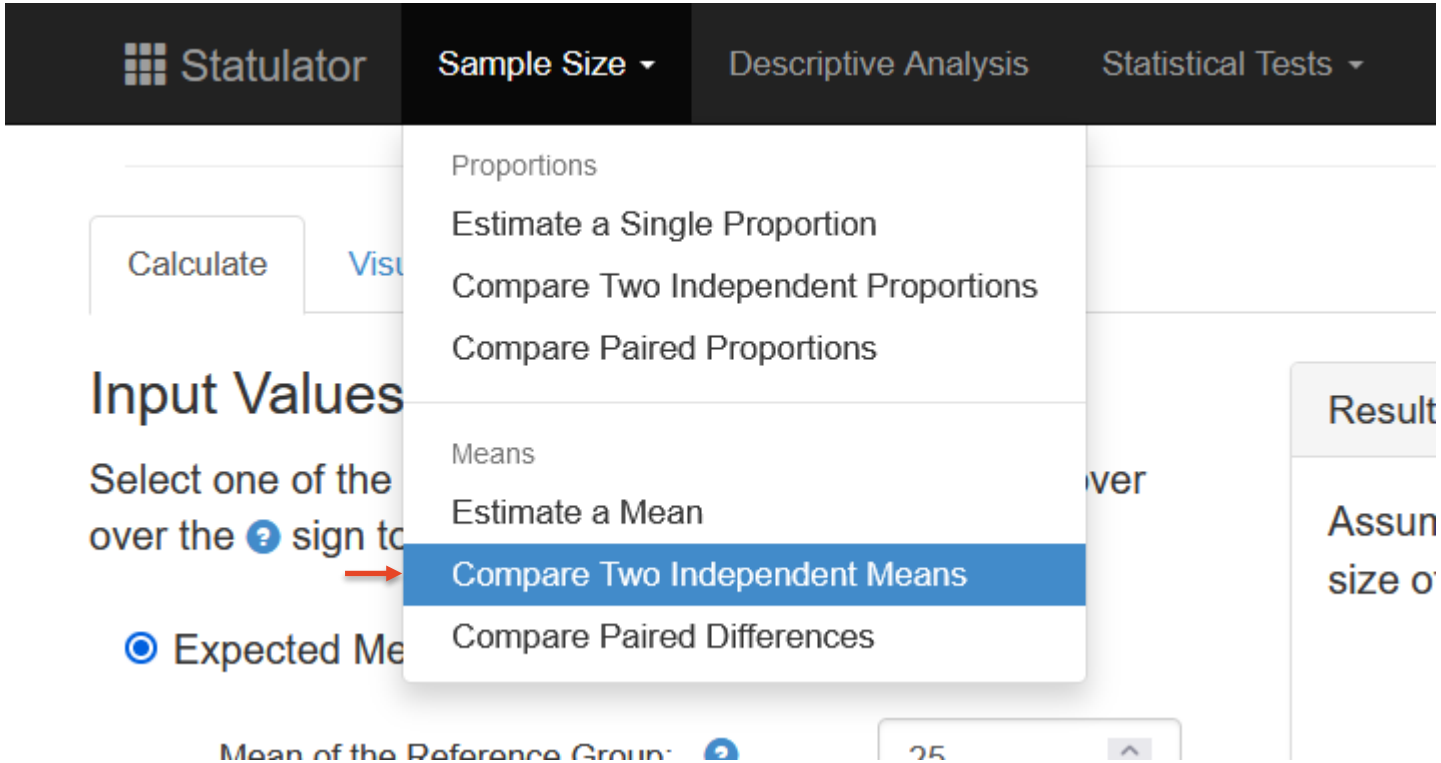
Example: Chicken Welfare – Bone density

- Step 1: We will use a t-test (assume normality)
- Step 2: α=0.05 and $1-\beta$=0.8 (leave values at their conventional levels)
- Step 3: Smallest Effect Size of interest is 10 mg/cm^3
- Step 4: Estimate the variance
  - We know from previous studies what the typical variation in bone density is for the control diet. We don't know about the treatment diet. We will use an estimate from the control diet of SD=20 mg/cm^3
- Assume we will have equal size groups, n1=n2

A. Difference between two means

Step 5: Statulator

Select Sample Size -> Compare Two Independent Means



A. Difference between two means

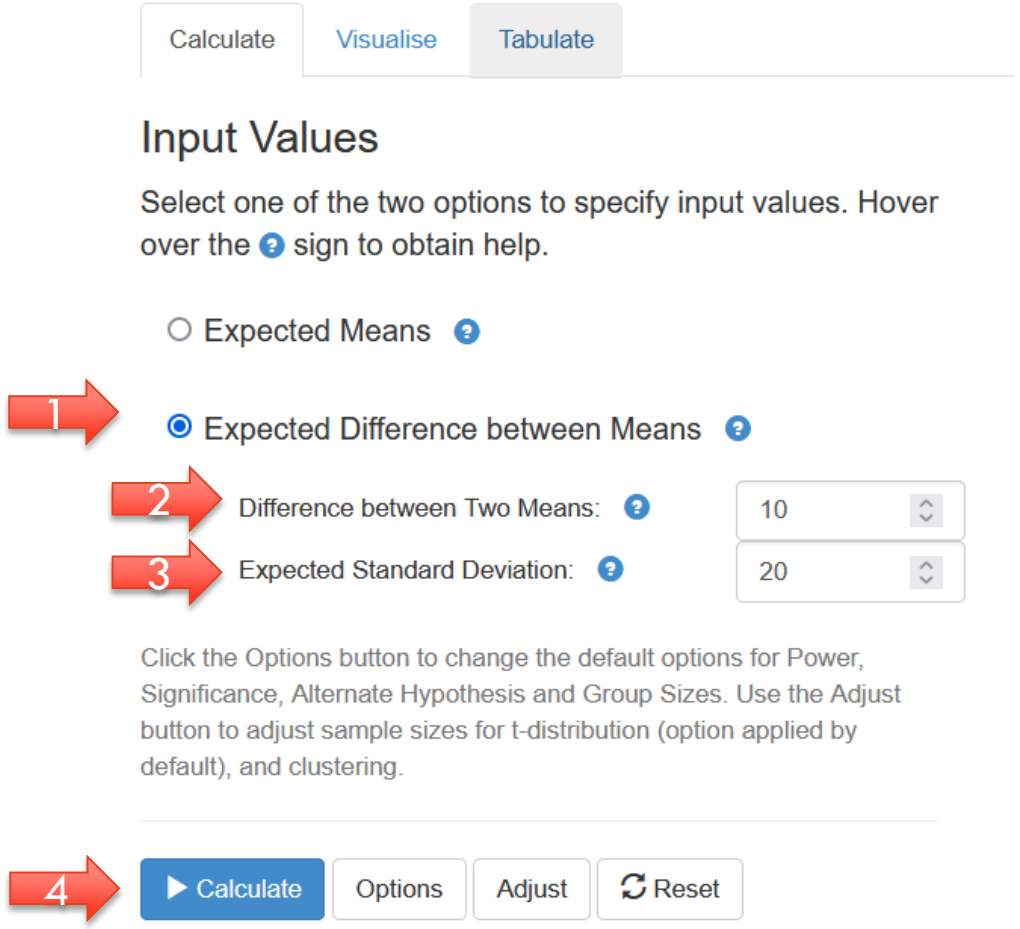
Step 5: Calculate the minimum sample size

- Put all the information into Statulator

– Choose expected difference between the means

– Difference between Two Means = 10 mg/cm^3
– Expected Standard Deviation = 20 mg/cm^3
– Leave all other Options at their defaults (80% power, 5% alpha)

– Click “Calculate”



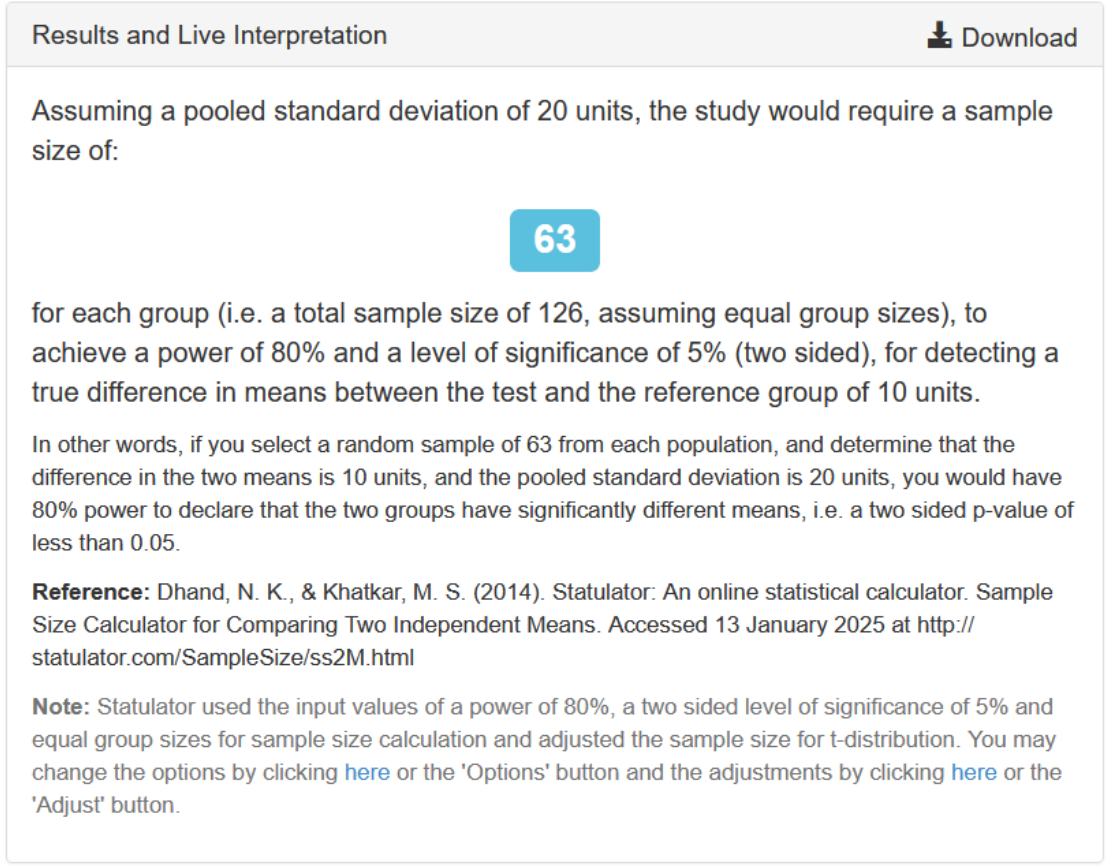
A. Difference between two means

Step 5: Calculate the minimum sample size

- Group sample sizes are N1=63, N2=63

- Statulator provides a plain-English explanation of the calculation. Relevant paragraphs can be included in your grant or ethics application

- Also a good opportunity to check that Statulator has done the calculation you think it has (in this case a two-sided test, using a t-distribution)



A. Difference between two means

Example: Chicken Welfare – Bone density

Step 6: Explore scenarios

Power Analysis

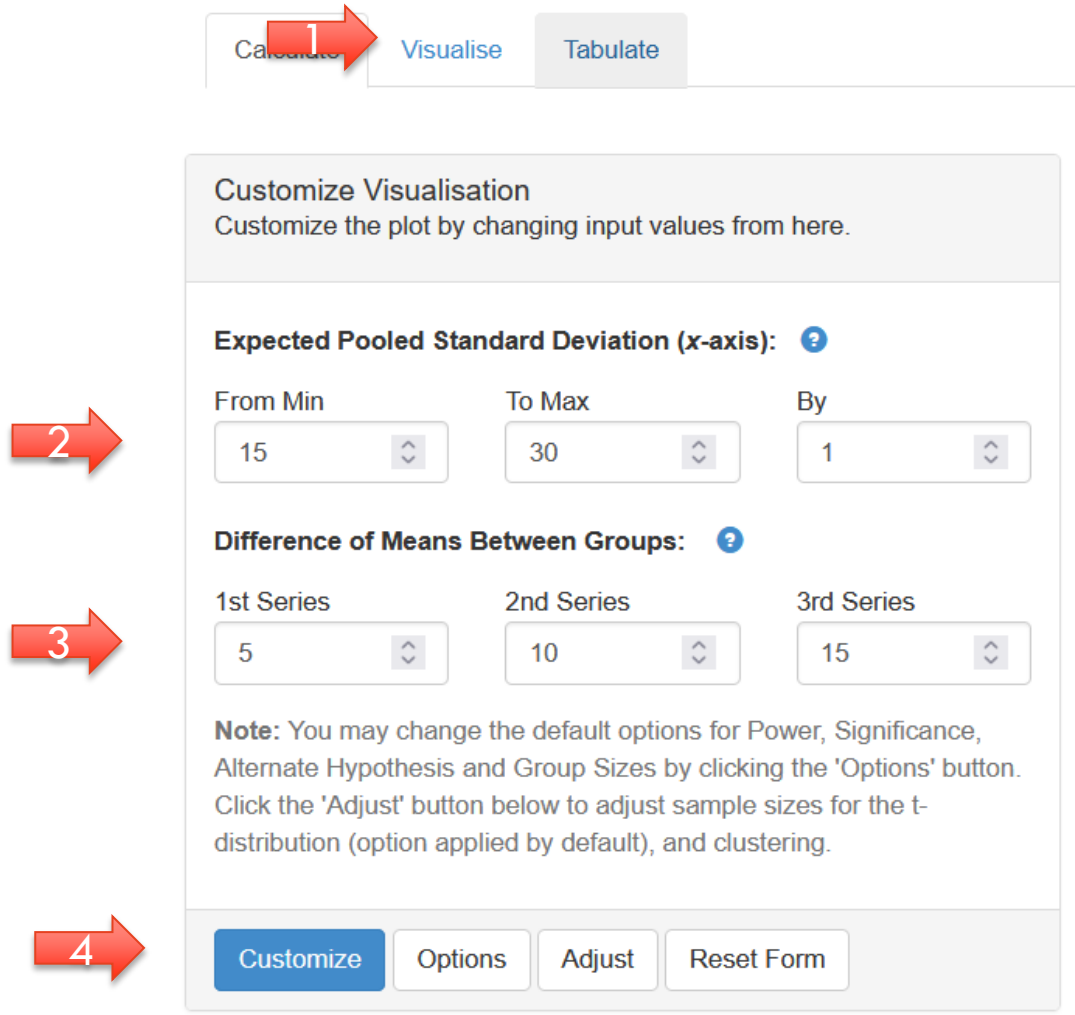
- It is advisable to explore some different scenarios to incorporate uncertainty in our inputs

- Consider how much your within study standard deviation could vary from your point estimate
  - Our estimate is SD = 20 mg/cm^3 (expected)
  - Possible min value = 15 mg/cm^3 (optimistic)
  - Possible max value = 30 mg/cm^3 (pessimistic, conservative)

- Consider how much the difference between the groups (effect size) could vary from our point estimate
  - Our estimate is 10 mg/cm^3
  - Possible min value = 5 mg/cm^3 (pessimistic, conservative)
  - Possible max value = 15 mg/cm^3 (optimistic)

A. Difference between two means

- Click the visualise tab
- Enter the ranges into Statulator and click ‘Customize’



A. Difference between two means

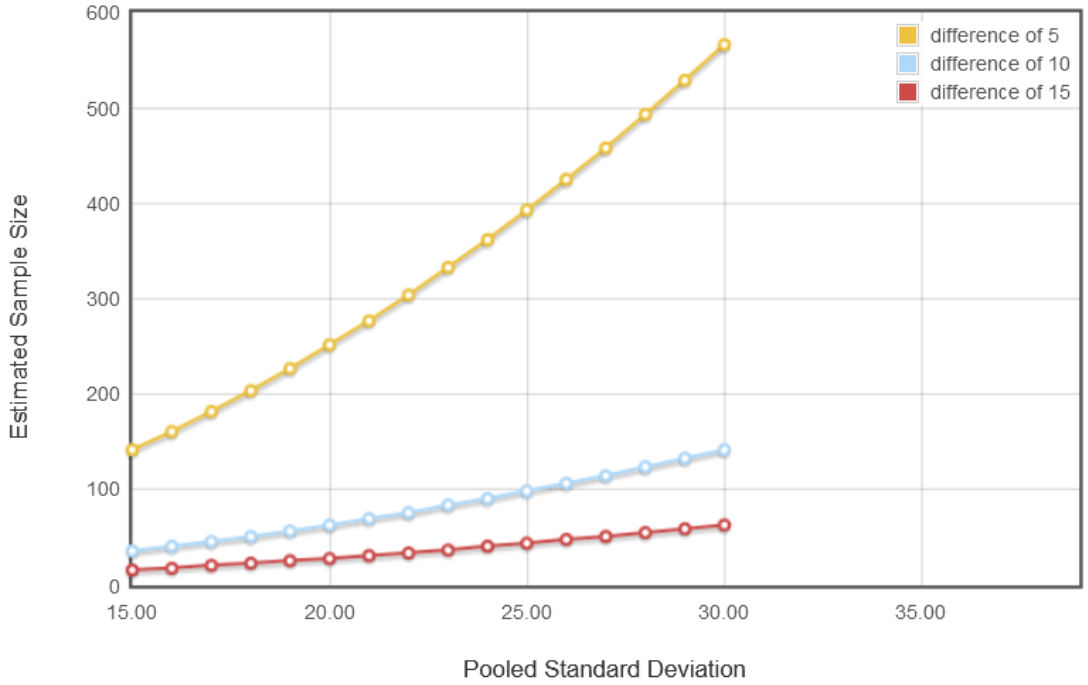
- We get a plot showing how the specified scenarios affect the required sample size.
- Although useful, for some this is not a complete power analysis and you may need additional plots
- For plots of sample size vs. power and sample size vs. detectable effect we can use Power and Sample Size (PS)

Visualisation

This is a plot of sample sizes for a range of pooled Standard Deviations and for three values of Difference of means between groups. Customize the plot by changing input values from the 'Customize Visualisation' panel.

Note that the sample sizes are displayed for only one of the two groups.

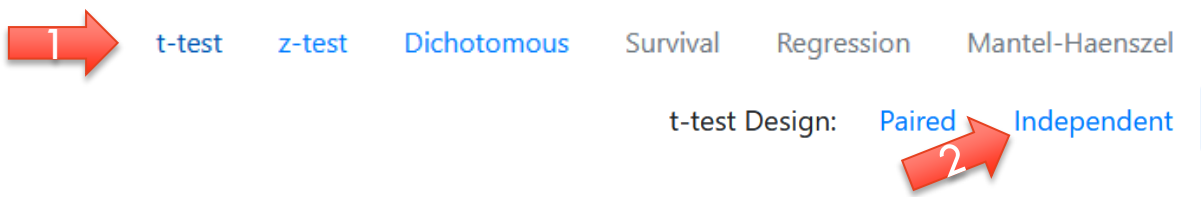
Download Figure



Note: Statulator used the input values of a power of 80%, a two sided level of significance of 5% and equal group sizes for sample size calculation and adjusted the sample size for t-distribution. You may change the options by clicking here or the 'Options' button and the adjustments by clicking here or the 'Adjust' button.

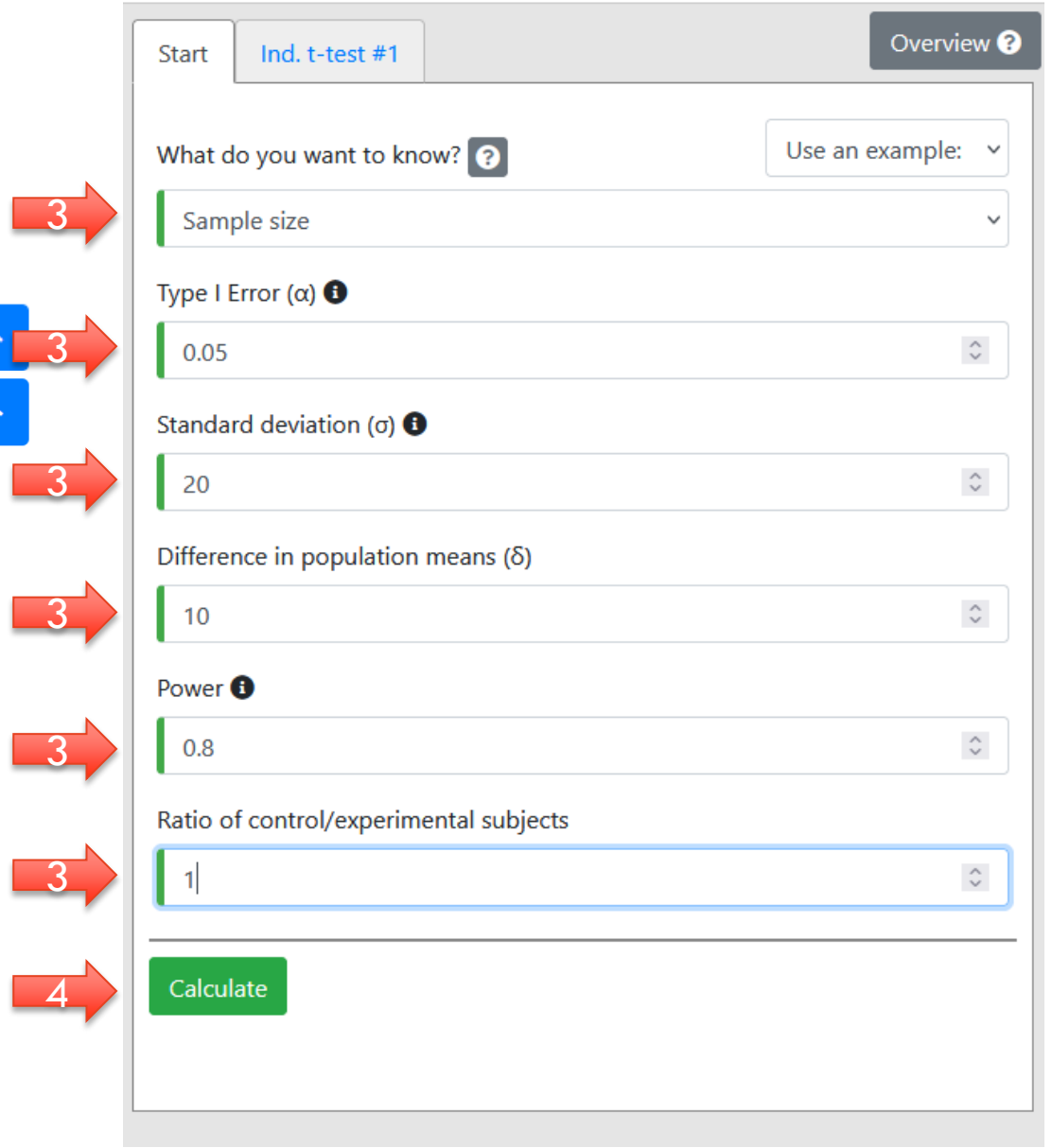
A. Difference between two means

- Choose t-test from the top naviation bar and then 'Independent'



- Under 'What do you want to know?' choose 'Sample size'

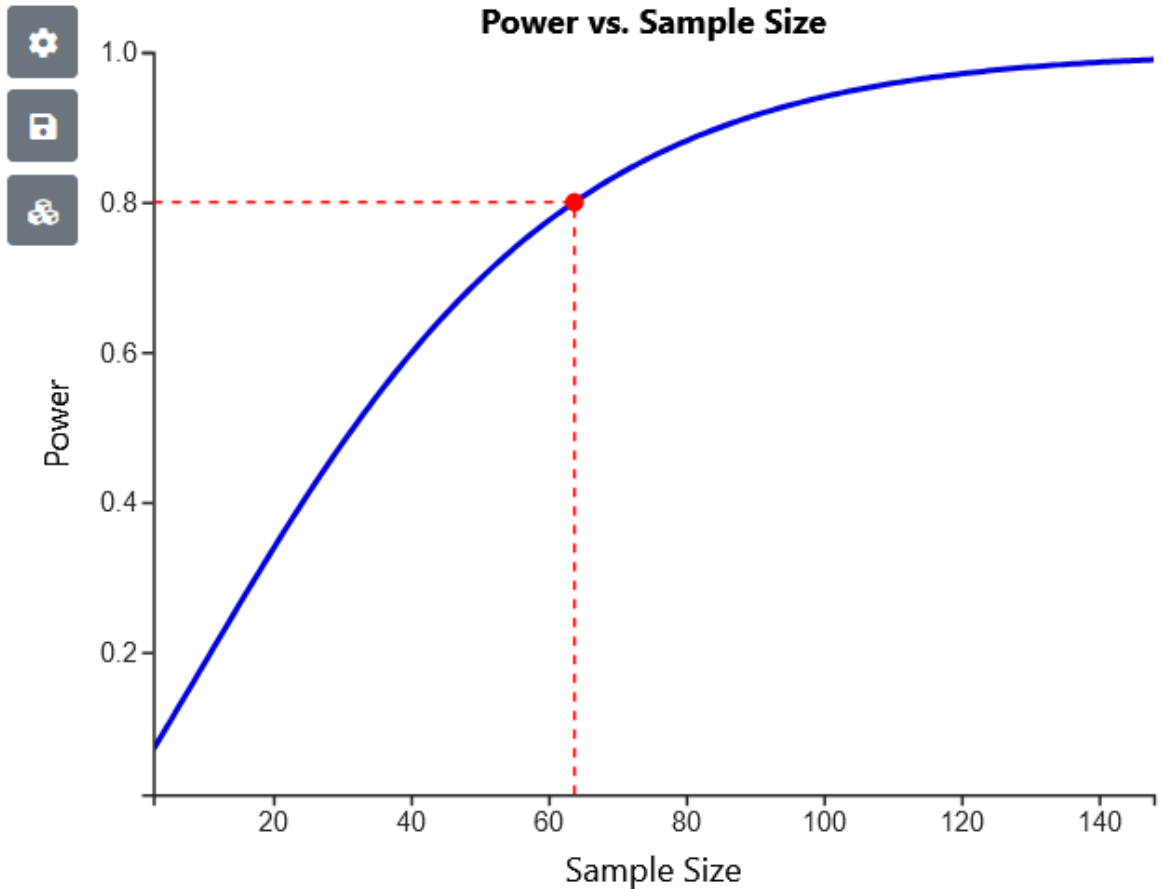
- Enter the same inputs as you did in Statulator (including Statulator defaults) and click Calculate



A. Difference between two means

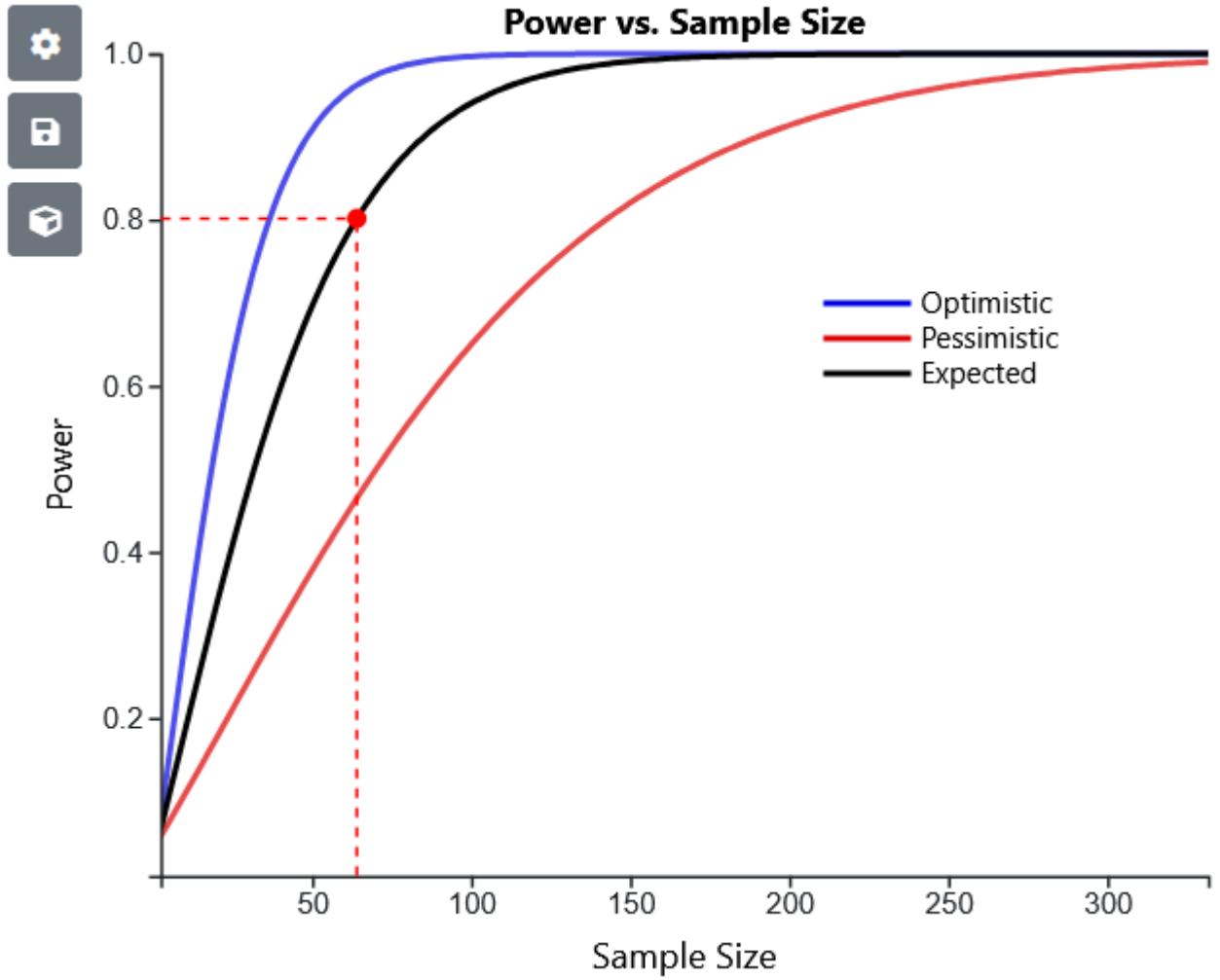
- When we click calculate, we get n1=n2=64 per group, and an interpretation paragraph
- We also see a series of **power analysis** plots that are customisable
- Change the 'output' drop down to Power to see plots with Power on the y-axis
- The top-left plot shows the amount of power for a range of sample sizes
- The sample size needed to achieve our desired power of 80% is highlighted with the red point

We are planning a study with 64 experimental subjects and 64 control subjects. In a previous study the response within each subject group was normally distributed with standard deviation 20.00. If the true difference in the experimental and control means is 10.00, we will be able to reject the null hypothesis that the population means of the experimental and control groups are equal with probability (power) 0.80. The Type I error probability associated with this test of this null hypothesis is 0.05.



A. Difference between two means

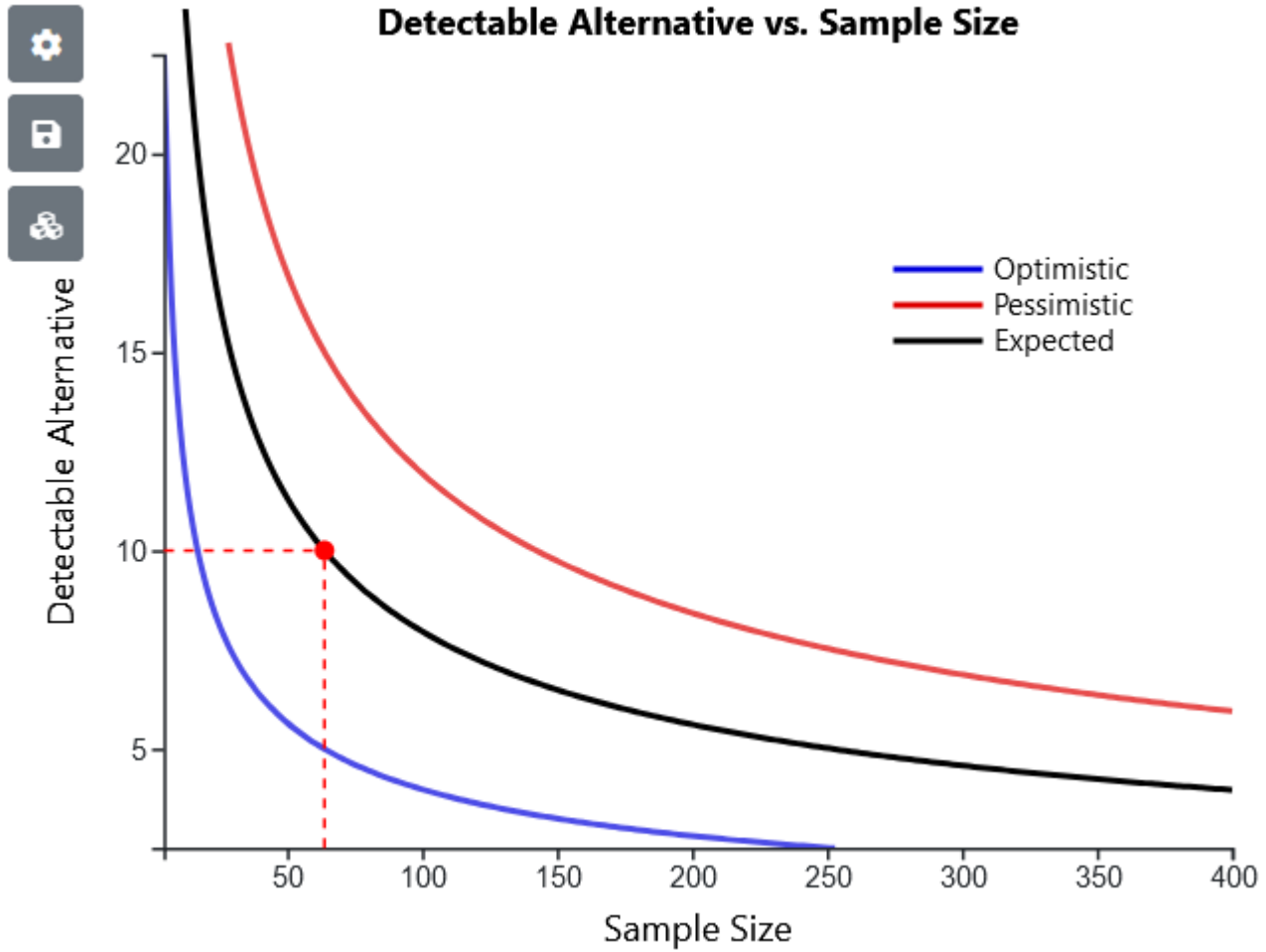
- To this plot we can add two more lines to evaluate our other scenarios for variance
- We evaluate a ‘pessimistic’ scenario (30 mg/cm3) and an ‘optimistic’ scenario (15 mg/cm^3)
- If we read across at 80% (desired) power, we get an idea of how large our sample size would need to be in each scenario



A. Difference between two means

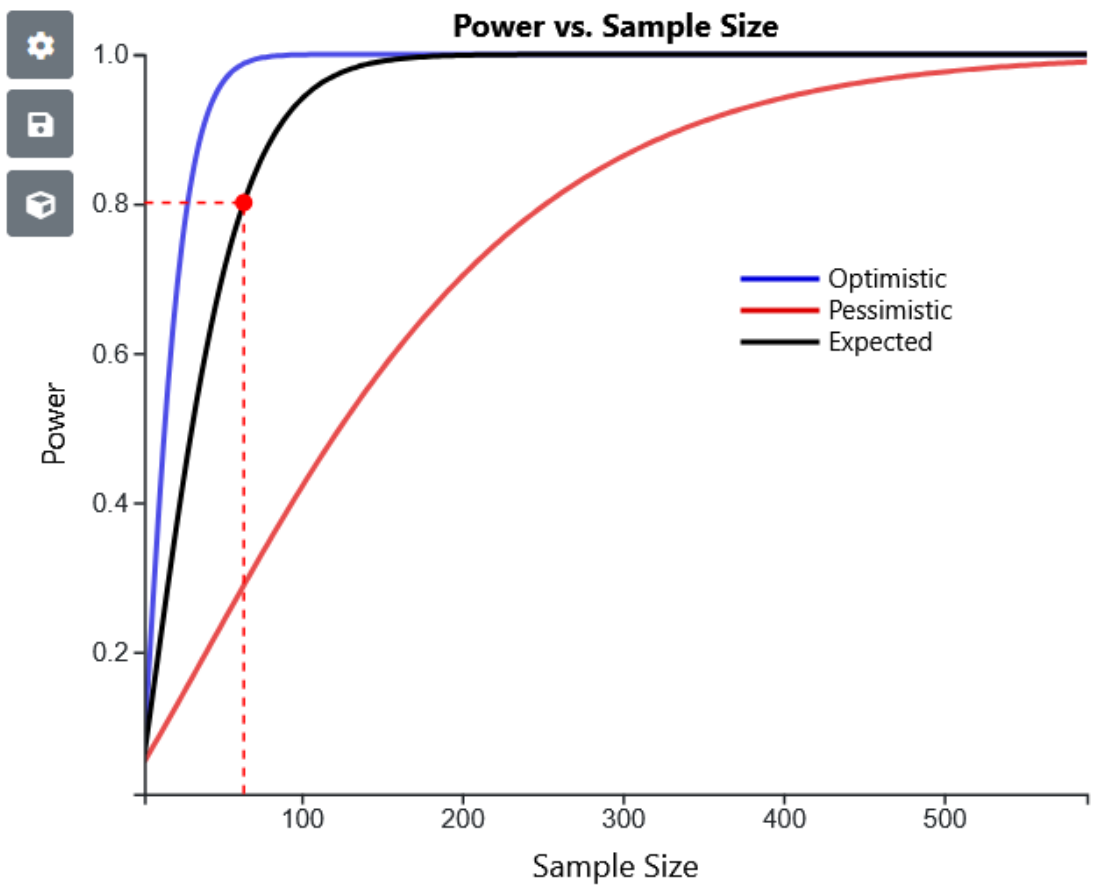
- Another very useful plot is a 'sensitivity' plot, showing the sample size vs. the minimum detectable effect size for a given power (in this case 80%)

- This plot shows that for our chosen sample size (of 64 per group):
  - A group difference of 10 will achieve 80% power in the 'Expected' scenario
  - A group difference of only 7.5 in the 'Optimistic' scenario
  - A group difference of 15 would be needed to achieve 80% power for the pessimistic scenario



We can also evaluate our effect size scenarios

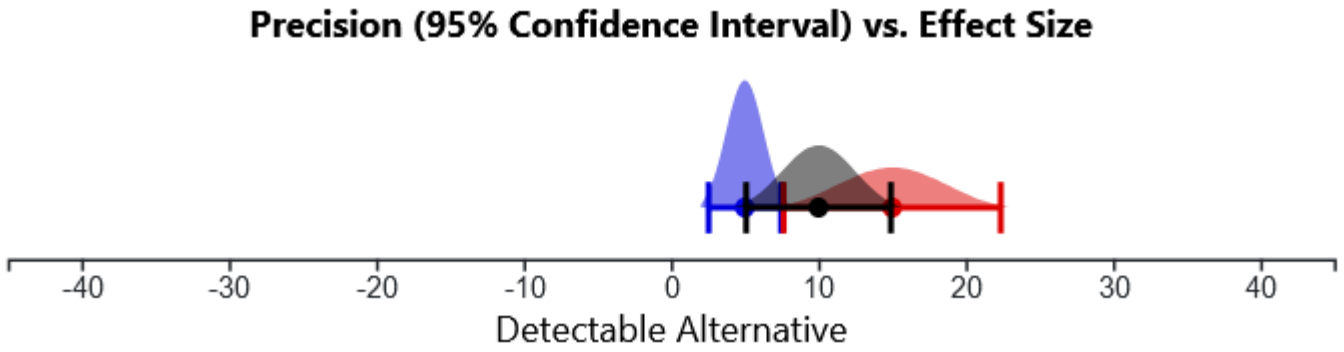
- We can also investigate our optimistic and pessimistic scenarios for effect size
- 3 scenarios for variance x 3 scenarios for effect size = 9 different scenarios, which is too many lines for this graph (only five different colours are supported)
- Instead, we will fix the variance at our expected 20 mg/cm^3, and evaluate the three effect-size scenarios



A. Difference between two means

- We also see an interesting plot that shows the point estimate, confidence interval, and distribution of sample mean differences we should expect if the true mean difference(s) in the population are exactly as we predict

- While this plot isn't as useful for exploring different scenarios, it is a good reminder of the connection between precision of estimates and hypothesis testing we will eventually perform (recall that the effect size is half the width of the confidence interval)



B. Difference between two means (Mann-Whitney)

The Mann-Whitney U test is a non-parametric version of the t-test for a difference in means. It is based on ranks (also called Wilcoxon rank sum)

This is used when the data are not approximately normally distributed (could be highly skewed), or the underlying distribution is not normal (could be ordinal).

Often used for ordinal data from surveys using Likert response items.

The values of the two groups are combined and ranked. The values are then divided back into the groups and the mean of the assigned ranks for each group is calculated and compared.

The test doesn't use the information about the size of the effect.

B. Difference between two means (Mann-Whitney)

Example: Happiness Survey

You want to measure happiness using the Lyubomirsky & Lepper scale. Each item response ranges from 1 (unhappy) to 7 (happy). The score is the sum of 4 items, so the range is 4~28.

A pilot study on two groups produced the following results that can be used for the power calculation (Mean and SD)

	Values		Ranks	
	Single	Married	Single	Married
	12	20	3	1
	11	15	4	2
	10	9	5	6
	6	8	8	7
Mean	9.8	13.0	5	4
SD	2.6	5.6		

B. Difference between two means (Mann-Whitney)

Example: Happiness Survey

You want to apply it to different groups of people (e.g. single vs married) to see if there is a difference in scores.

What is a meaningful difference?

Let's suppose that a minimum difference of 4 points (average of 1pt difference per item) is the smallest effect size of interest.

B. Difference between two means (Mann-Whitney)

Example: Happiness Survey

So, what are our first 4 steps?

Step 1:	Determine experiment type and statistical test	Two group comparison using Mann-Whitney Group 1: Single Group 2: Married
Step 2:	Set α and $1-\beta$	0.05 and 0.8
Step 3:	Set the smallest effect size of interest	4 points
Step 4:	Estimate the variance	SD1=2.6, SD2=5.6

B. Difference between two means (Mann-Whitney)

Example: Happiness Survey

Sample size calculation

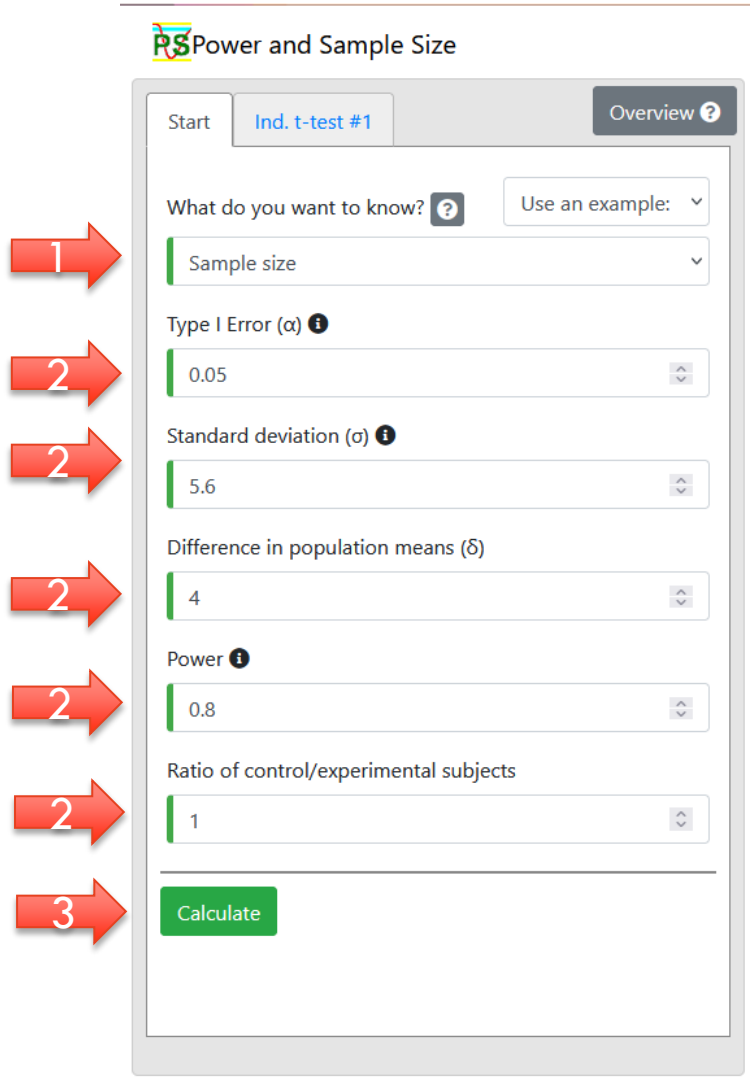
Heuristic method

"Do the calculations as if performing the corresponding parametric test (i.e. the t-test), then add 15% to the sample size."

B. Difference between two means (Mann-Whitney)

Example: Happiness Survey

- t-test>Independent
- Enter Difference in population means=4
- Enter the larger SD (Standard deviation=5.6)
- Enter other inputs
- Click ‘calculate’



B. Difference between two means (Mann-Whitney)

Example: Happiness Survey

- N=32 per group
- Add 15% for non-parametric. N=32x1.15 = 36.8 round up to 37

How and why to calculate within-group standard deviation

- In this example we had two groups with quite different variance/SDs: SD1=2.6, SD2=5.6
- The most conservative choice is to choose the within group standard deviation as the larger SD group (SD2 in this example)
- Unequal variances do not usually cause a problem when group sizes that are equal
- If the group sizes are equal, we can calculate a pooled standard deviation easily using Cohen's formula:

$$\sigma' = \sqrt{\frac{\sigma_1^2 + \sigma_2^2}{2}}$$

- In this example: $\sqrt{\frac{2.6^2+5.6^2}{2}} = 4.37$, when input results in a sample size of 20 per group
- Add 15% for non-parametric: 20 x 1.15 = 23 per group

B. Difference between two means (Mann-Whitney)

Theoretical approach

Statistical procedures can be compared according to their efficiency.

One test is more efficient than another if it requires fewer observations to obtain a given result.

The relative efficiency of two tests is the ratio of their efficiencies.

With smaller sample numbers, parametric tests are often more efficient than non-parametric tests although they approach equal efficiency with larger sample sizes.

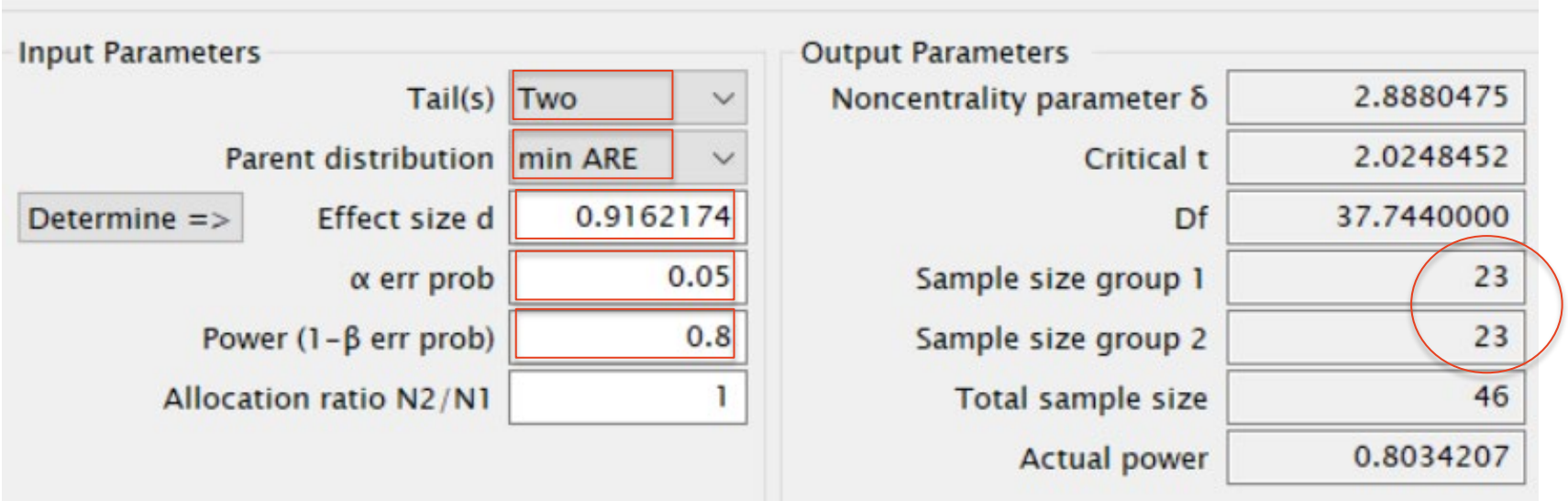
The Asymptotic Relative Efficiency (ARE) is the limit of the relative efficiencies as the sample size increases. It can be calculated or set and is used in the sample size calculation, along with the effect size.

It can be shown that the minimum ARE for these two tests is 0.864.

B. Difference between two means (Mann-Whitney)

Example: Happiness Survey

- Under "Tests" select "Means" and then the option:
- "Two independent groups: Wilcoxon (non-parametric)
- Use the same values as before:
- Two tails, α=0.05 and Power=0.80, group means and SDs.
- Select Parent distribution = "min ARE"
- Calculate sample size >> N=23 per group



C. Difference between two proportions

Example: Happiness survey

The survey scores could also be analysed as proportions by considering how many report a value above a threshold (say >14 means "happy")

Singles group P1 = proportion of subjects respond "happy"

Married group P2 = proportion of subjects respond "happy"

Effect size: Say we want to find a minimum difference in proportions of P1-P2=0.1 What sample size is required?

- We also need to estimate the two proportions. Let's first assume that there will be maximum variance (p=0.50)
- Try using P1=0.55 and P2= 0.45. As p = (p1 + p2)/2.

C. Difference between two proportions

Example: Happiness survey

What are our first 4 steps this time?

Step 1:	Determine experiment type and statistical test	z-test for proportions
Step 2:	Set α and $1-\beta$	0.05 and 0.8
Step 3:	Set the smallest effect size of interest	0.10
Step 4:	Estimate the variance	p1=0.55, p2=0.45

Note: The variance estimate comes from the proportion estimates and is calculated for you.

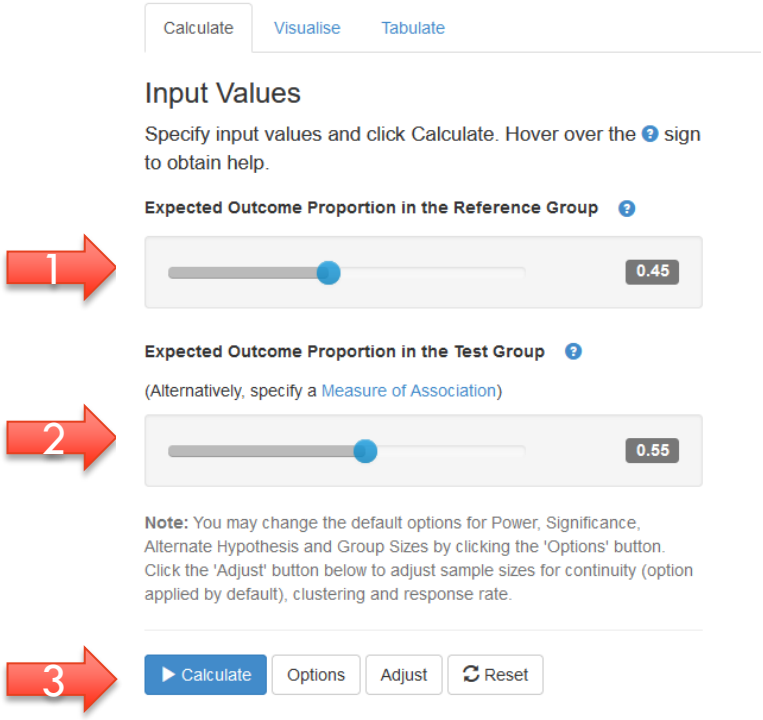
Variance = p(1-p).

C. Difference between two proportions

Example: Happiness survey

Step 5: Statulator

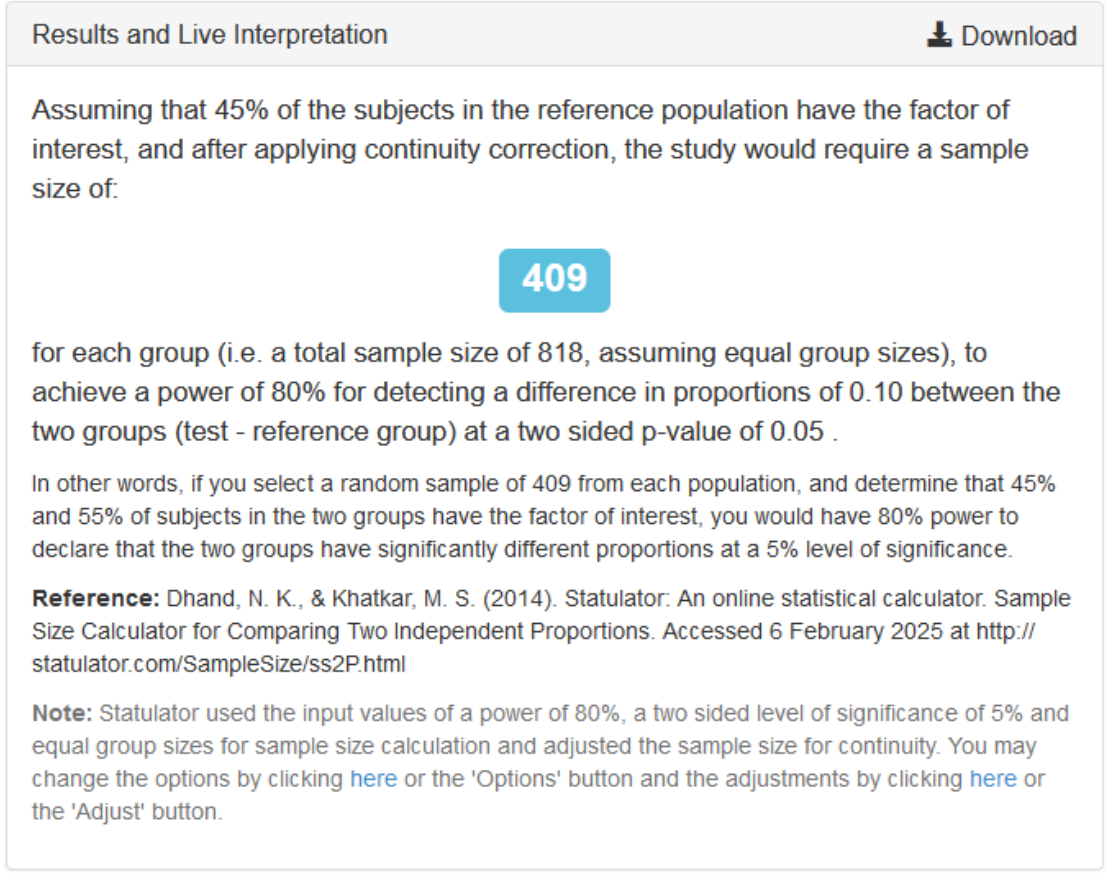
Select Sample Size -> Compare Two Independent Proportions



C. Difference between two proportions

- The same outcome (happiness) that required 32 per group, with a binary variable requires 409 per group
- That's a lot of people

- Tip from the consulting room:
  - With a binary outcome each subject provides only one [computer] bit of information: 0 or 1
  - If you do have an option to use the underlying scale as your outcome, your required sample size for a given power will often be drastically smaller


Results and Live Interpretation

Download

Assuming that 45% of the subjects in the reference population have the factor of interest, and after applying continuity correction, the study would require a sample size of:

409

for each group (i.e. a total sample size of 818, assuming equal group sizes), to achieve a power of 80% for detecting a difference in proportions of 0.10 between the two groups (test - reference group) at a two sided p-value of 0.05 .

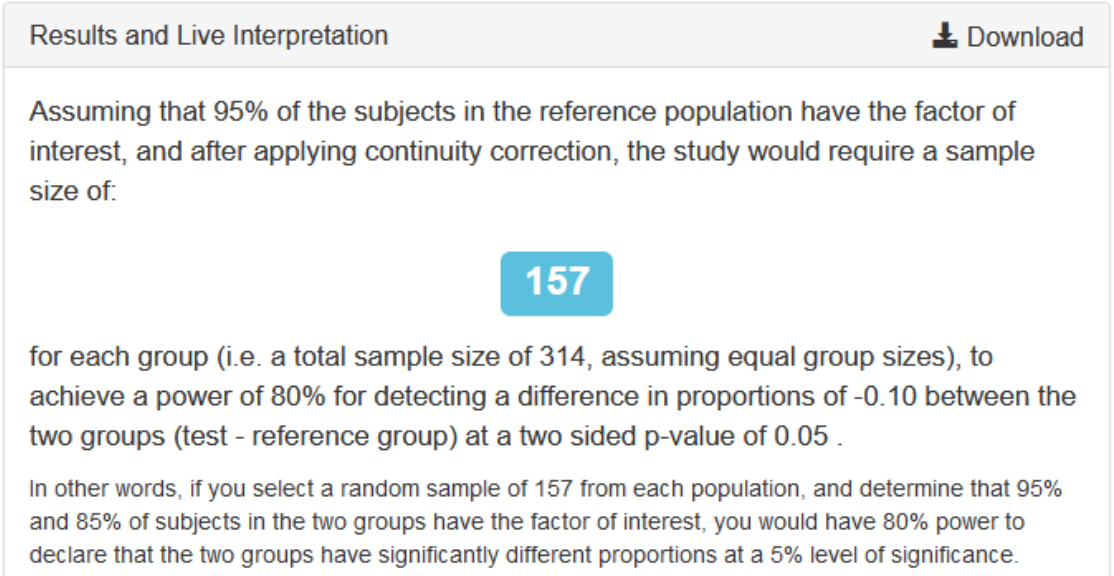
In other words, if you select a random sample of 409 from each population, and determine that 45% and 55% of subjects in the two groups have the factor of interest, you would have 80% power to declare that the two groups have significantly different proportions at a 5% level of significance.

Reference: Dhand, N. K., & Khatkar, M. S. (2014). Statulator: An online statistical calculator. Sample Size Calculator for Comparing Two Independent Proportions. Accessed 6 February 2025 at http://statulator.com/SampleSize/ss2P.html

Note: Statulator used the input values of a power of 80%, a two sided level of significance of 5% and equal group sizes for sample size calculation and adjusted the sample size for continuity. You may change the options by clicking here or the 'Options' button and the adjustments by clicking here or the 'Adjust' button.

C. Difference between two proportions

- We deliberately chose the proportions that would produce the maximum variance in the outcome for a given group difference (of 10%)
- What if we collected more pilot data, and saw that our expected proportions were closer to: p1= 95% and p2 = 85%
- The required sample size shrinks to 157 per group, which a lot less but still far more than when using the scale outcome directly



C. Difference between two proportions

There are multiple options when comparing independent proportions

G*Power provides a total of 4 options for power calculations for proportions with independent groups:

- Inequality, z-test (used in Happiness intervention example)
- Inequality, Fisher's Exact test
- Inequality, Unconditional exact
- Inequality with offset, Unconditional exact

The Fisher's Exact test should be used when sample sizes are going to be small (say $n_1 p_1 \leq 5 \; or \; n_2 p_2 \leq 5$)

– The Fisher's Exact result for the Happiness example is shown on the next slide for your reference

C. Difference between two proportions

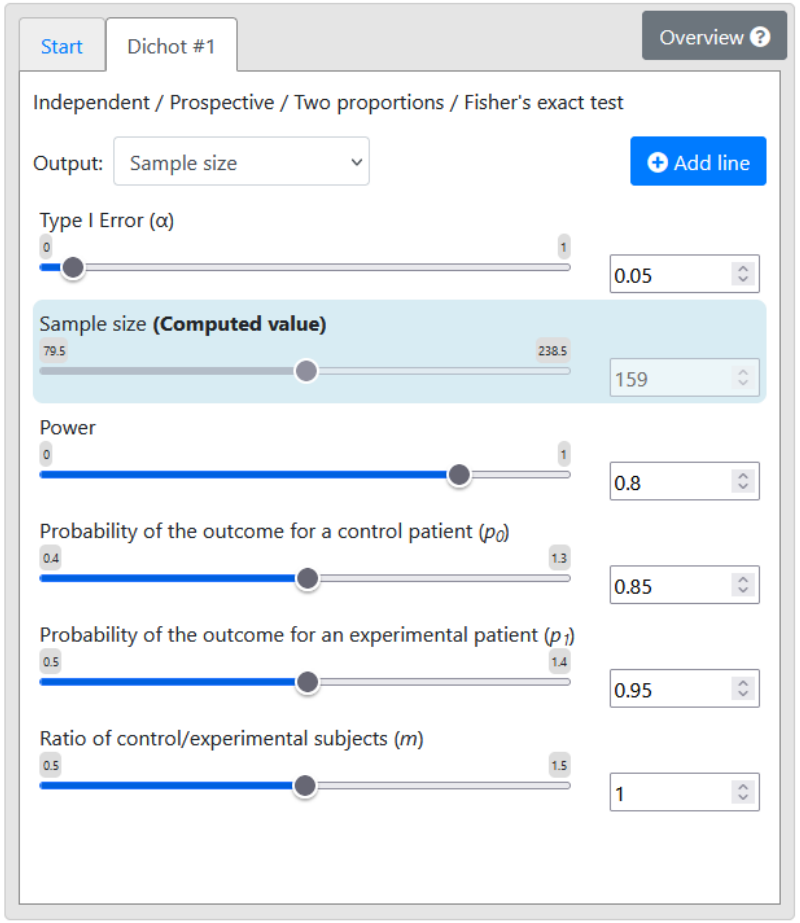
Example: Happiness survey

Step 6: Use the Fisher's Exact test to get the sample size with

P1=0.85 and P2=0.95

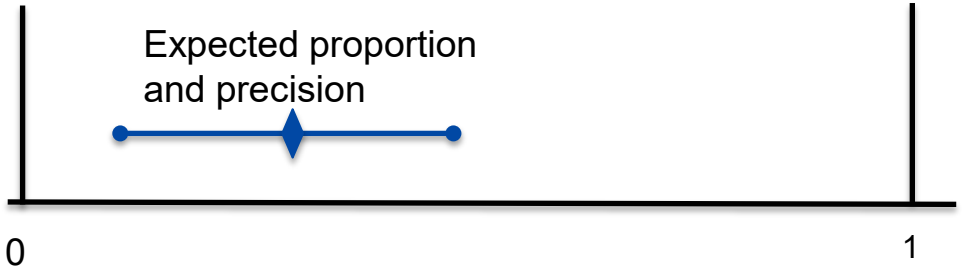
Fisher's Exact suggests 159 subjects per group.

Not quite the same result as the z-test, but very close



D. Estimation of a single proportion

- A common sample size calculation that does not involve a hypothesis test
- Proportions are encountered often:
  - Prevalence of disease in a population
  - Proportion of people failing a screening test
  - Sensitivity and specificity of a diagnostic test
- The sample size calculation is to estimate a proportion to a given level of precision
- Precision in this context is the width of the confidence interval (usually 95% confidence interval)



D. Estimation of a single proportion

- Lets continue the happiness example, but estimate the prevalence of happiness in the general population
- We need to generalise our workflow for sample size to a given precision
  1. Determine experiment type and statistical test **or population property to estimate**
  2. Set α [~~and $1-\beta$~~]
  3. Set the smallest effect size of interest **or required precision**
  4. Estimate the variance
  5. Calculate the minimum sample size
  6. Explore scenarios

Step 1:	Proportion of happy people
Step 2:	0.05 (for 95% confidence interval)
Step 3:	0.10 (or 10%)
Step 4:	Use maximal variance at p=0.5

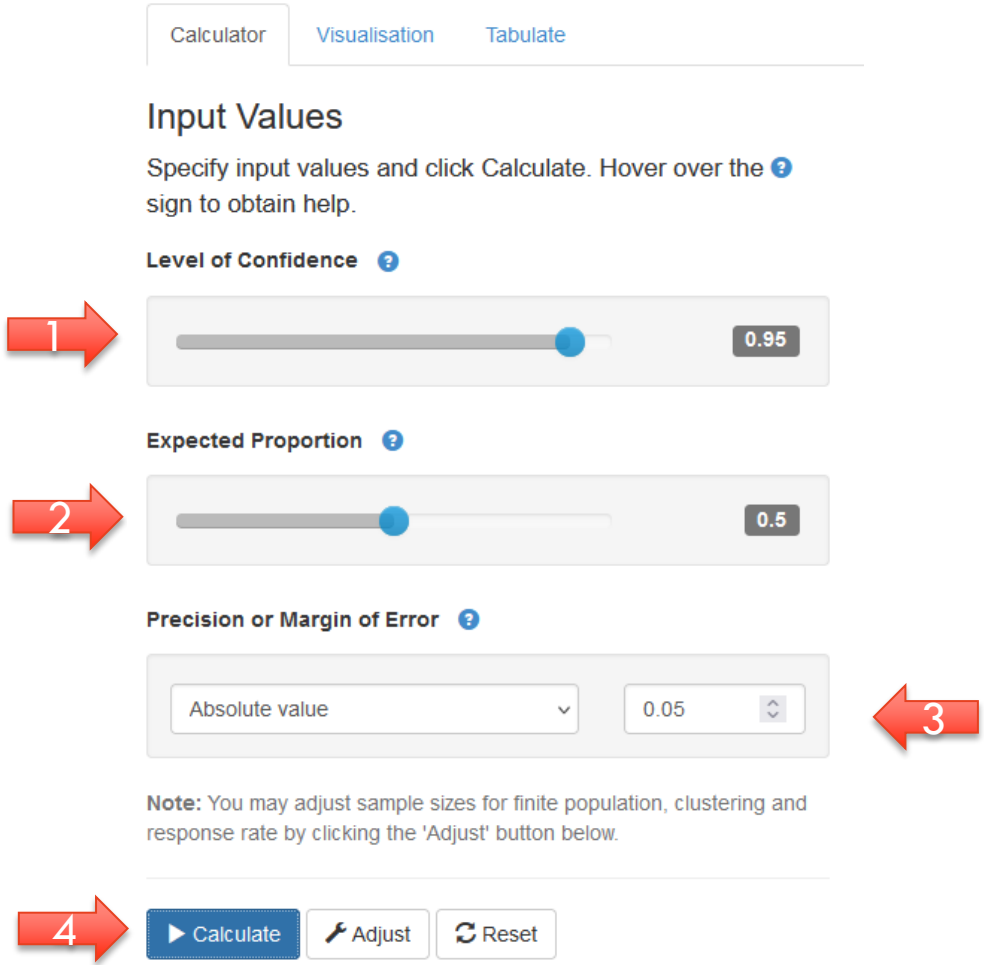
D. Estimation of a single proportion

Example: Happiness survey

Step 5: Statulator

Select Sample Size -> Estimate a Single Proportion

Step 1:	Determine population property	Proportion of happy people in general population
Step 2:	Set α	0.05 (95% CI)
Step 3:	Set the smallest effect size of interest	0.10 CI width
Step 4:	Estimate the variance	Maximal at p=0.5

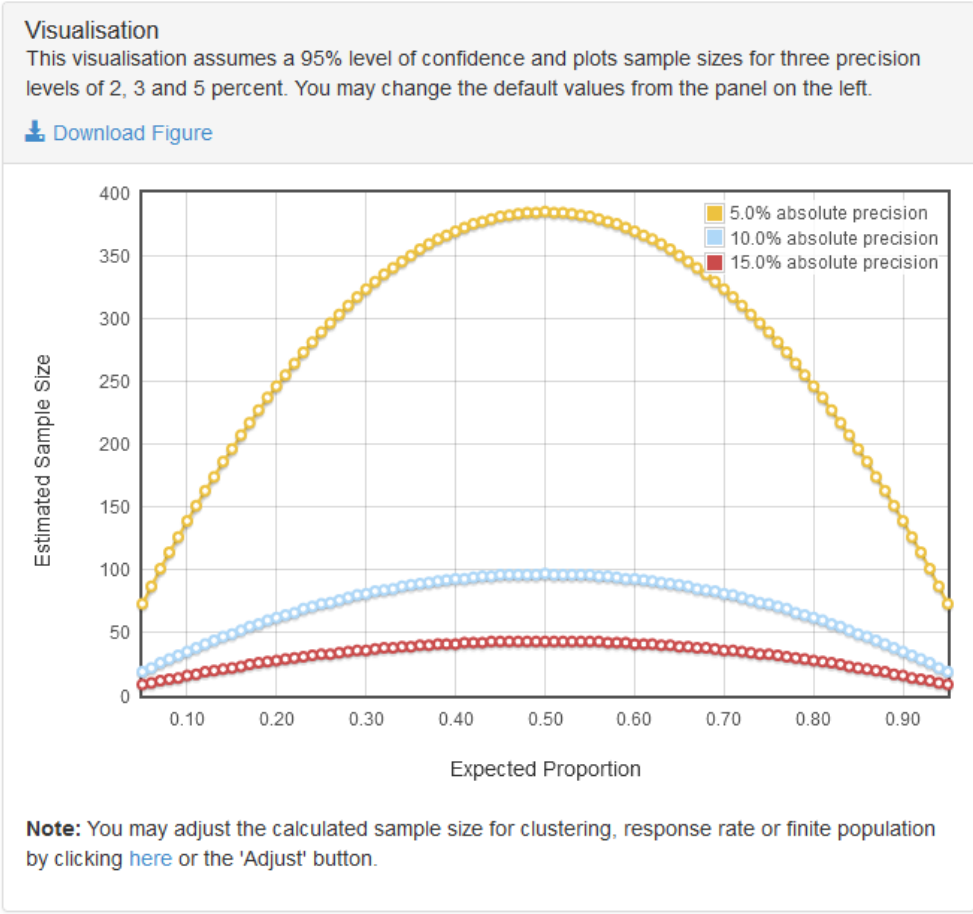


The precision, specified here as an 'absolute value' is half the width of the desired confidence interval

D. Estimation of a single proportion

- The sample size required is 385 for a 10% wide confidence interval

- Again, the maximal variance is when the p=0.5, so if we have a better idea of our expected p, the required sample size may be less than this



Handling dropout

Tip from the consulting room:

- It is common, especially in clinical trials to inflate the required sample size to account for an expected rate of dropout
- One pitfall is to inflate the minimum sample number (N) by the expected dropout rate (W). After dropouts this would not achieve the required sample size of completers
- Instead we need to use the formula for N** for the inflated sample size that will restore the required sample size of completers after dropouts

$$N^{**} = \frac{N}{1 - W}$$

Worked example for N=100, and W=15%

Wrong:

100 x 115% = 115 x 15% = 17.25 so 18 dropouts and 97 completers

Right:

100 / 85% = 117.64, so 118 required

118 x 15% = 17.7 dropouts and 100 completers

Handling multiple testing

- Multiple testing corrections, must be incorporated into your power analysis
- The easiest way to incorporate these is to change the alpha in the sample size calculation to match the reduced 'per test' alpha needed to achieve the Family Wise Error Rate (FWER) alpha
- For Bonferroni correction, the 'per test' alpha is simply the FWER alpha divided by the number of tests
- For more complex multiple testing corrections, you may need to chose a different alpha to input
- You also need to consider your study goals, and your analysis plan. Feel free to have a consult to discuss further

Power Analysis for other designs

Statulator scope:

- Two group [in]dependent means, [in]dependent proportions, single proportions

Power and Sample Size (PS) scope:

- As above but also OR under prospective or retrospective designs, precision of confidence intervals

G*Power scope:

- Most of above but also ANOVA, Correlation, Linear Regression, Logistic Regression

Refer to the manual for details. Search for "Gpower manual" online or find it here

Effect Sizes for other designs

Effect size for ANOVA

G*Power uses the standardised effect size; Cohen's f

f is related to the partial eta squared

$$\eta^2 = \frac{f^2}{(1 + f^2)}$$

Partial eta squared is often reported in the ANOVA table output

Recall that this F test is rarely used to power a study. Instead choose the two groups that are most similar and power a t-test to detect the difference between them (after accounting for multiple testing in post-hoc).

Effect size for other designs: Use a wide variety of effect size measures

Power Analysis for other designs

From simple designs to complex designs

So far we have considered power analysis for simple designs where the mathematical calculations are tractable and rely on a limited set of assumptions regarding the data to be obtained.

As design complexity increases, it becomes more difficult or perhaps impossible to find an analytical solution to calculate power.

When no formula exists:

- First option – determine sample size for a simplified version of the study design and extrapolate this to the more complex design
- Second option – Use a simulation method (that does not rely on formulae)

Power Analysis by simplification

A couple of examples

ANOVA: Choose the two groups that are most similar and power the post-hoc test to detect the expected difference between them. All other post-hocs and the F test should have sufficient power.

Multiple regression:

- For a categorical factor of interest choose the factor of interest and power to detect the difference as a t-test (the same as a univariate model). The addition of covariates should only increase the power of your actual analysis
- If you have an idea of the variance explained by your factor of interest and the residual variance you can use the linear multiple regression module of G Power

Linear mixed models:

- If you have a repeated measures experiment, power the study as if you only had one measurement. The addition of repeated measures should only increase the power of your actual analysis

Switch to simulation methods for complex study designs where analysis of a simplified design is not sufficiently rigorous.

Power Analysis – by simulation

Simulation based power estimation

- Simulate (many) data sets
- Analyse each data set and test for statistical significance
- Calculate the proportion of simulations with significant p values

$$Power = \frac{significant\ simulations}{all\ simulations}$$

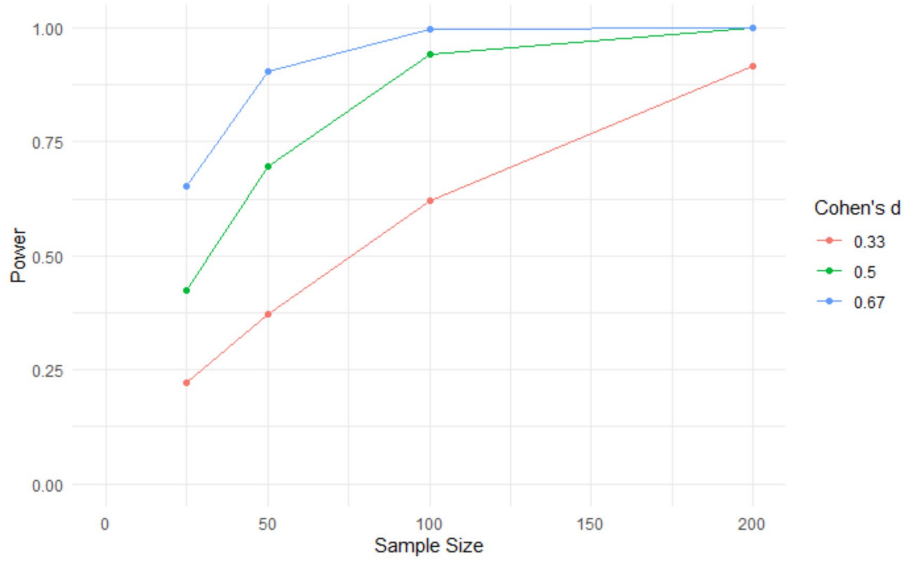
- The 'trick' is to set the parameters of the simulation in a sensible, realistic way

https://link.springer.com/article/10.3758/s13428-021-01546-0

Power Analysis – by simulation

Example A: Chicken Welfare - bone density (difference between two means)

- Simulation in R using package "paramtest"
- Results for this simple simulation will be very similar to those obtained from G*Power.
- See R Markdown files for details



Generalised linear models

- You can use the package **simr**
- Specify the model as you would for analysis
- **Simr** then simulates data from that model

Software for Power Analysis

Free and Open Source software

- R /R Studio:
  - Base R has functions covering basic proportions, t-tests, etc.
  - Package "pwr" has 9 functions covering proportions, t-tests, ANOVA, chi-square and correlations
  - Package "epiR" has 23 functions covering many statistics including AUC, sensitivity and specificity
  - Package "paramtest" basic power calculations by simulation
  - Package "mixedpower" for generalised linear mixed models
  - Package "simr" simulation based power calculations for mixed models

- Online calculators such as www.powerandsamplesize.com and https://sample-size.net/ and https://www.statulator.com/SampleSize/
- G*Power is a dedicated (free) program
- Make your own in Excel! (for example see Lakens, D. (2013). Calculating and reporting effect sizes to facilitate cumulative science: A practical primer for t-tests and ANOVAs. Frontiers in Psychology, 4:863. doi:10.3389/fpsyg.2013.00863)

Software for Power Analysis

Proprietary $$ software

- Packages such as STATA, SPSS and SAS include a calculator
- GraphPad have “StatMate” separate to Prism
- PASS by NCSS dedicated software esp. for medical research

Challenge Questions

1. Power calculation

Your power calculation gives a sample size of n=20 per group, with α =0.05 and power = 0.8.

The Type I error rate is:

a. 80%
b. 20%
c. 5%
d. 2.5%

Challenge Questions

1. Power calculation

Your power calculation gives a sample size of n=20 per group, with α =0.05 and power = 0.8.

The Type I error rate is:

a. 80%
b. 20%
c. 5%
d. 2.5%

Challenge Questions

2. Effect Size

If you halve the smallest effect size of interest in your power calculation, to achieve the same statistical power…

a. you will need twice the sample size
b. you will need half the sample size
c. you will need 4 times the sample size
d. you will increase the Type I error

Challenge Questions

2. Effect Size

b) If you halve the smallest effect size of interest in your power calculation…

a. you will need twice the sample size
b. you will need half the sample size
c. you will need 4 times the sample size
d. you will increase the Type I error

inverse square relationship between effect size and n

$$n = \frac{2\,(Z_{\alpha/2} + Z_{1-\beta})^2}{\left(\frac{\mu_1 - \mu_2}{\sigma}\right)^2}$$

Challenge Questions

3. Sample Size calculation

You are planning an experiment involving measuring the weight gain of 2 groups of foals (horses). One group is assigned Treatment A and the other Treatment B.

What hypothesis test would you use?

* _____ or maybe * _____

What pieces of information do you need to determine the sample size?

1	
2	
3	group balance (eg n1=n2)
4	
5	
6	one-tailed or two-tailed test

Challenge Questions

3. Sample Size calculation

You are planning an experiment involving measuring the weight gain of 2 groups of foals (horses). One group is assigned Treatment A and the other Treatment B.

What hypothesis test would you use?

* t-test or maybe * Mann Whitney (non-parametric)

What pieces of information do you need to determine the sample size?

1	alpha level (Type I error rate)
2	power level (1- Type II error rate)
3	group balance (eg n1=n2)
4	standard deviation (within group variance)
5	minimum weight increase (min. effect size of interest)
6	one-tailed or two-tailed test

Challenge Questions

4. Error Types

The types of error that may result from a hypothesis test are analogous with the errors that a jury might make when deciding on guilt or innocence of a defendant.

If the jury wrongly convicts, what type of error has occurred?

_____

If the jury acquits the defendant, but she was actually guilty, what type of error has occurred?

_____

"Beyond reasonable doubt" is a high standard of proof. It should result in a low _____error rate

Challenge Questions

4. Error Types

The types of error that may result from a hypothesis test are analogous with the errors that a jury might make when deciding on guilt or innocence of a defendant.

If the jury wrongly convicts, what type of error has occurred?

Type I

If the jury acquits the defendant, but she was actually guilty, what type of error has occurred?

Type II

"Beyond reasonable doubt" is a high standard of proof. It should result in a low Type I error rate

Questions?

Your turn…

Power calculation references

This workshop was originally developed by Jim Matthews, from the Statistical Consulting Unit at Sydney Informatics Hub

- **G*Power** http://www.psychologie.hhu.de/arbeitsgruppen/allgemeine-psychologie-und-arbeitspsychologie/gpower.html
- **NCSS PASS Statistical software** https://www.ncss.com/software/pass/
- **Causal Evaluation** https://www.causalevaluation.org/power-analysis.html
- **Epi Tools for disease prevalence (by AUSVET)** http://epitools.ausvet.com.au/content.php?page=SampleSize
- **Demidenko (Dartmouth) for logistic regression** https://www.dartmouth.edu/~eugened/power-samplesize.php
- **National Institutes of Health (NIH – USA) for cluster randomised trials** https://researchmethodsresources.nih.gov/SampleSizeCalculator.aspx
- **UCSF Clinical and Translational science institute** (Survival for clinical research) http://www.sample-size.net/sample-size-survival-analysis/
- **Lakens, D. Open Science Framework** https://osf.io/ixGcd/

Power Analysis – library references

- **Cohen, Jacob. Statistical Power Analysis for the Behavioral Sciences.**
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  https://sydney.primo.exlibrisgroup.com/permalink/61USYD_INST/14vvljs/alma991005702359705106

- **Dattalo, Patrick. Determining Sample Size Balancing Power, Precision, and Practicality**
  Oxford: Oxford University Press, 2008. Print.
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  Boca Raton: CRC Press/Taylor & Francis, 2010. Print.
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- **Ryan, Thomas P., and Thomas P Ryan. Sample Size Determination and Power.**
  Somerset: John Wiley & Sons, Incorporated, 2013. Web.
  https://sydney.primo.exlibrisgroup.com/permalink/61USYD_INST/1367smt/cdi_askewsholts_vlebooks_9781118439203

Further Assistance at Sydney University

SIH

- Statistical Consulting website: containing our workshop slides and our favourite external resources (including links for learning R and SPSS)
- Hacky Hour an informal monthly meetup for getting help with coding or using statistics software
- 1on1 Consults can be requested on our website (click on the big red 'contact us' link)

SIH Workshops

- Create your own custom programmes tailored to your research needs by attending more of our Statistical Consulting workshops. Look for the statistics workshops on our training page.
- Other SIH workshops
- Sign up to our mailing list to be notified of upcoming training

Other

- Open Learning Environment (OLE) courses
- Linkedin Learning

A reminder: Acknowledging SIH

All University of Sydney resources are available to Sydney researchers **free of charge**. The use of the SIH services including the Artemis HPC and associated support and training warrants acknowledgement in any publications, conference proceedings or posters describing work facilitated by these services.

The continued acknowledgment of the use of SIH facilities ensures the sustainability of our services.

Suggested wording for use of workshops and workflows:

"The authors acknowledge the Statistical workshops and workflows provided by the Sydney Informatics Hub, a Core Research Facility of the University of Sydney."

We value your feedback

We want to hear about you and whether this workshop has helped you in your research. What **worked** and what **didn't work.**

We actively use the feedback to improve our workshops.

Completing this survey really does help us and we would appreciate your help! It only takes a few minutes to complete (*promise!*)

You will receive a link to the anonymous survey by email